FORECASTING

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Apa Arti Runtut Waktu?

- **Data runtut waktu (*time series*) merupakan data yang dikumpulkan, dicatat, atau diobservasi sepanjang waktu secara berurutan**
- **Periode waktu dapat tahun, kuartal, bulan, minggu, dan dibeberapa kasus hari atau jam.**
- **Runtut waktu dianalisis untuk menemukan pola variasi masa lalu yang dapat dipergunakan untuk:**

  (1) memprakirakan nilai masa depan dan membantu dalam manajemen operasi bisnis;

  (2) membuat perencanaan bahan baku, fasilitas produksi, dan jumlah staf guna memenuhi permintaan dimasa mendatang.

Mengapa Mempelajari Analisis Runtut Waktu?

Karena dengan mengamati data runtut waktu akan terlihat empat komponen yang mempengaruhi suatu pola data masa lalu dan sekarang, yang cenderung berulang dimasa mendatang

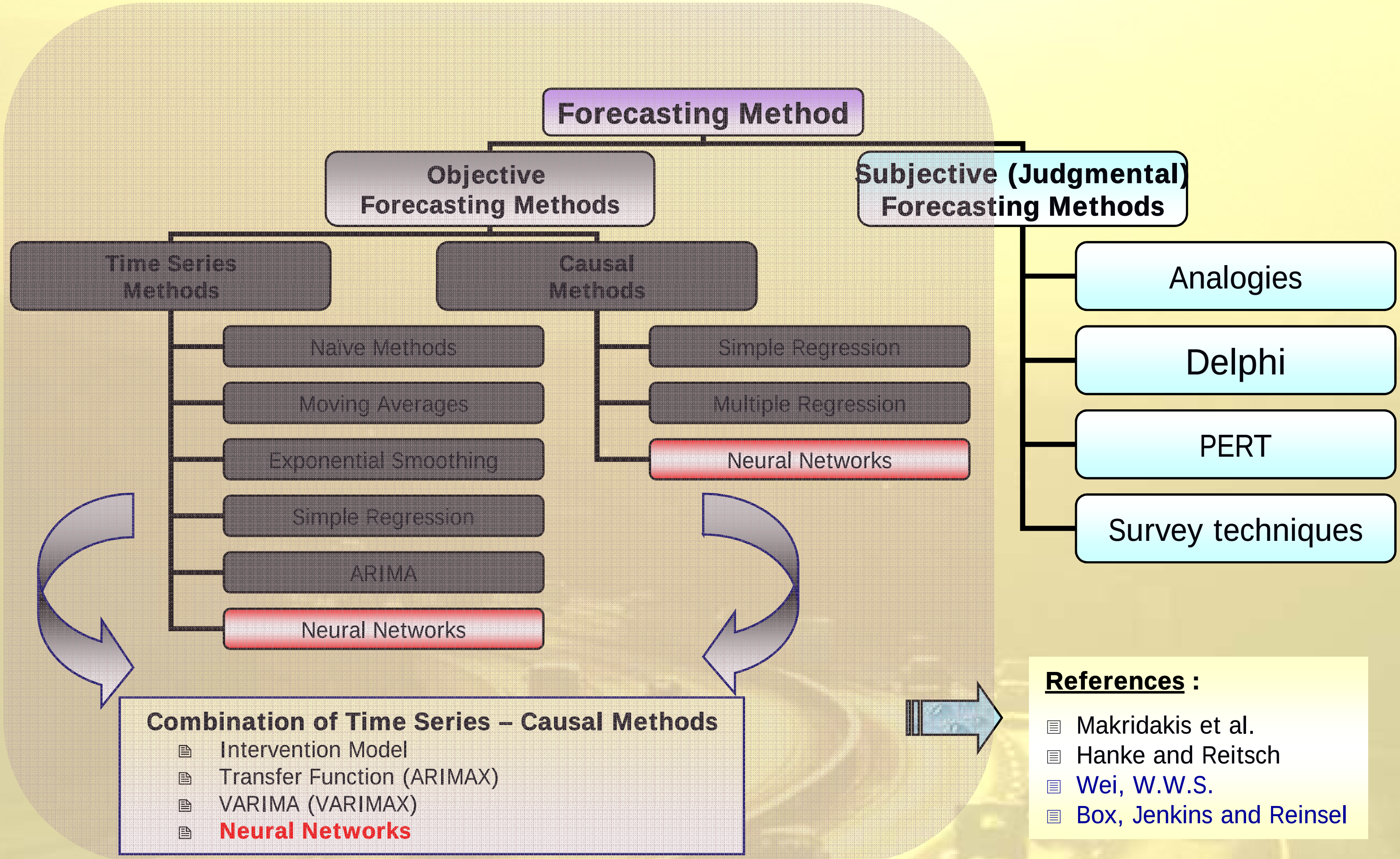
Teknik *Forecasting*

Pendekatan	**Basis**	**Teknik**	**Hasil**
Peramalan ekstrapolatif	Ekstrapolasi trend	Analisis rangkaian-waktu Teknik benang-hitam Teknik OLS Pembobotan eksponensial Transformasi data Metode katastrofi	Projeksi
Peramalan Teoretis	Teori	Pemetaan teori Analisis jalur Analisis Input-Output Pemrograman linier Analisis regresi Estimasi interval Analisis hubungan	Prediksi
Peramalan intuitif	Penilaian subjektif	Delphi konvensional Delphi kebijakan Analisis dampak-silang Penilaian kelayakan	Konjektur

Asumsi Peramalan Ekstrapolatif

1. Keajegan (*persistence*): Pola yang terjadi di masa lalu akan tetap terjadi di masa mendatang. Mis: jika konsumsi energi di masa lalu meningkat, ia akan selalu meningkat di masa depan.
2. Keteraturan (*regularity*): Variasi di masa lalu akan secara teratur muncul di masa depan. Mis: jika banjir besar di Jakarta terjadi setiap 16 tahun sekali, pola yg sama akan terjadi lagi.
3. Keandalan (*reliability*) dan kesahihan (*validity*) data: Ketepatan ramalan tergantung kepada keandalan dan kesahihan data yg tersedia. Mis: data ttg laporan kejahatan seringkali tidak sesuai dg insiden kejahatan yg sesungguhnya, data ttg gaji bukan merupakan ukuran tepat dari pendapatan masyarakat.

Klasifikasi Metode Peramalan …



References :

- Makridakis et al.
- Hanke and Reitsch
- Wei, W.W.S.
- Box, Jenkins and Reinsel

Klasifikasi Metode Peramalan : Ilustrasi Model Matematis . . .

Forecasting Method

Forecasting Methods

Su... tal)
Forecasting Methods

Examples :
$sales_{(t)} = f(sales_{(t-1)}, sales_{(t-2)}, \ldots)$

Examples :
$sales_{(t)} = f(price_{(t)}, advert_{(t)}, \ldots)$

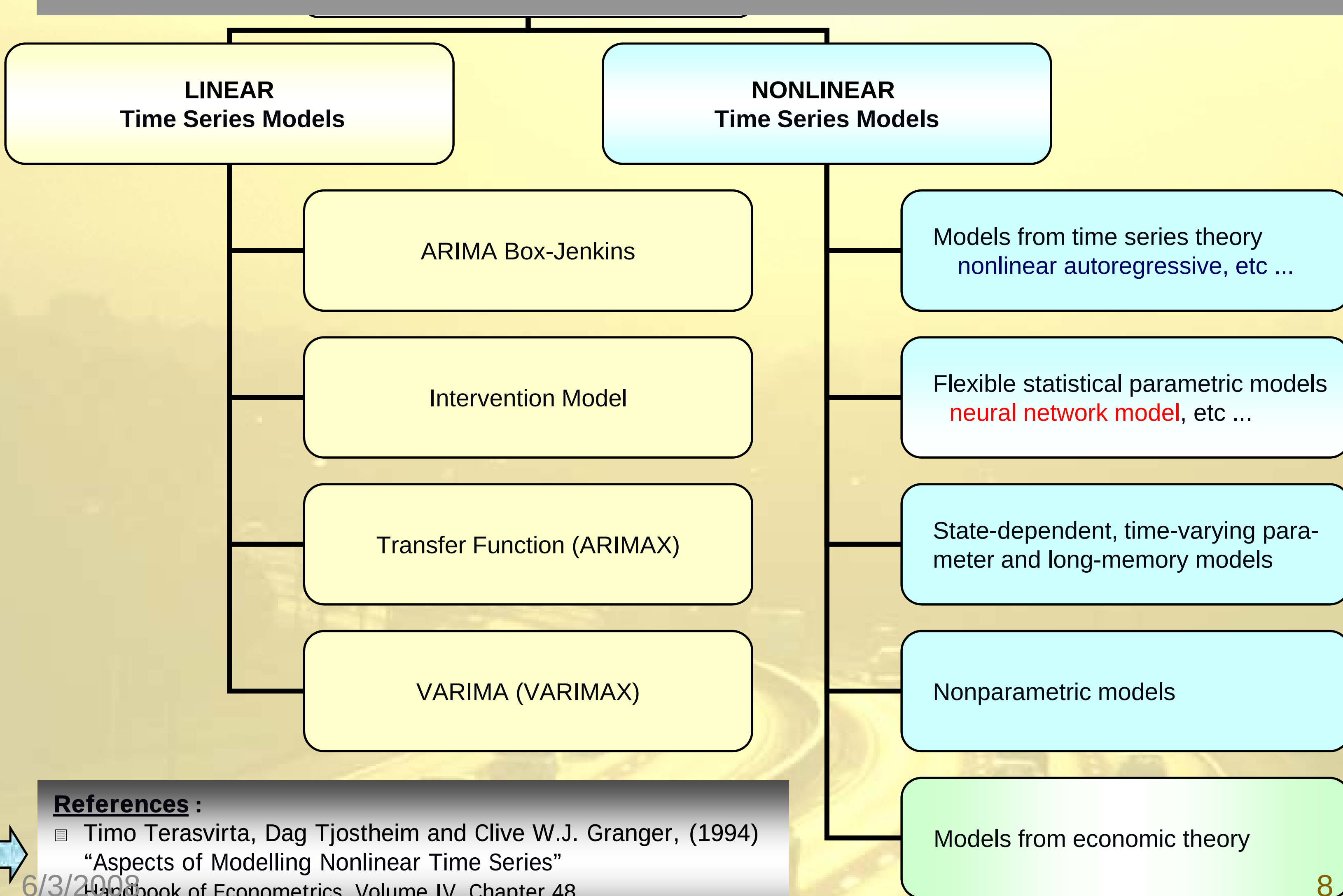
Combination of Time Series – Causal Methods
$Y_t = f(Y_{t-j}, j>0 ; X_{t-i}, i \; 0)$

Examples :
➨ $sales_{(t)} = f(sales_{(t-1)}, advert_{(t)}, advert_{(t-1)}, \ldots)$

Time Series M...
$Y_t = f(Y_{t-1}, Y_{t-2}, \ldots, Y_{t-k})$

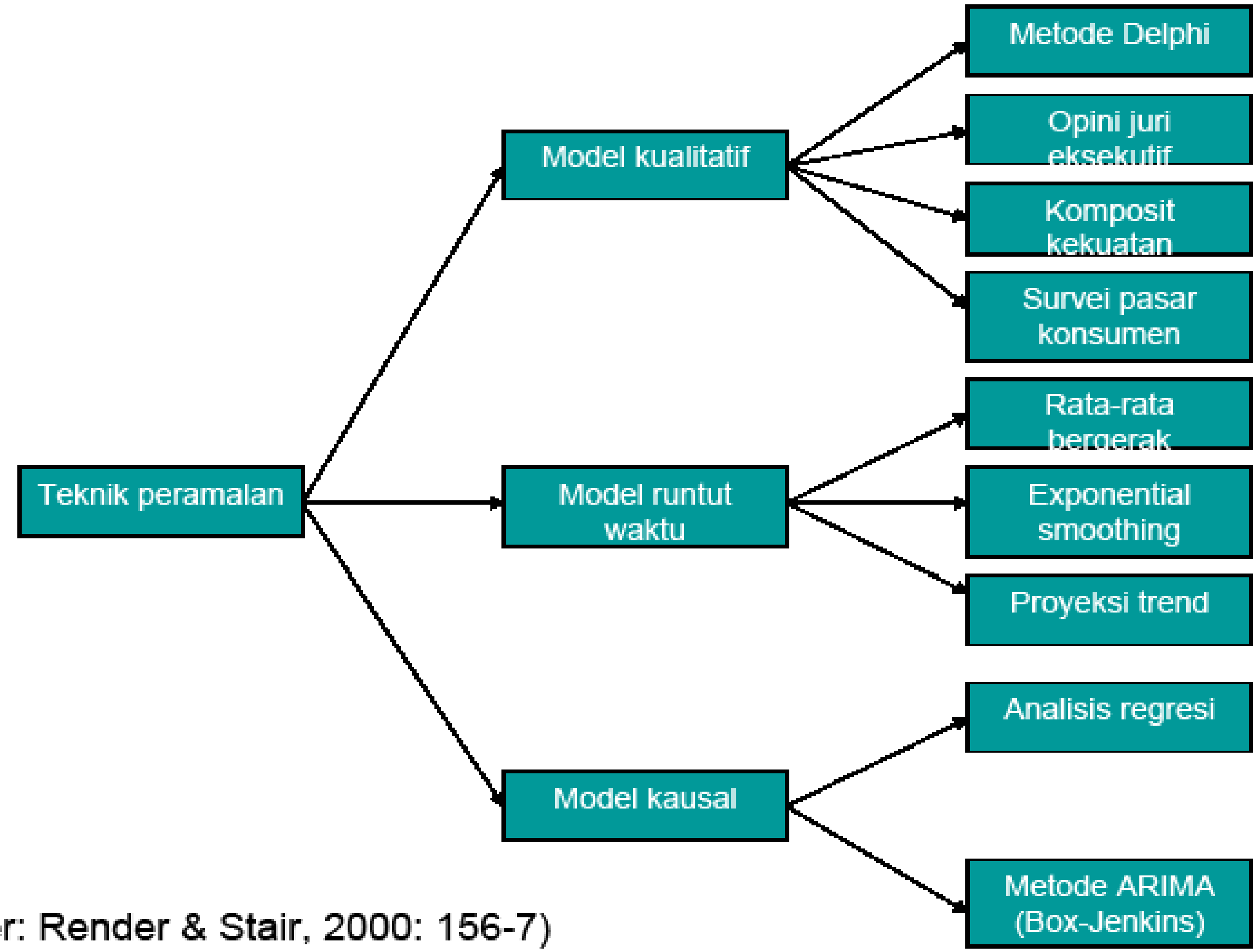
$Y_t = f(X1_t, X2_t, \ldots, Xk_t)$

Klasifikasi Model Time Series : Berdasarkan **Bentuk** atau **Fungsi** …



References :
- Timo Terasvirta, Dag Tjostheim and Clive W.J. Granger, (1994) "Aspects of Modelling Nonlinear Time Series"
  Handbook of Econometrics, Volume IV, Chapter 48.
  Edited by R.F. Engle and D.I. McFadden

Jenis Teknik Peramalan



Sumber: Render & Stair, 2000: 156-7)

Model Runtut Waktu

- Model runtut waktu berusaha untuk memprediksi masa depan dengan menggunakan data historis
- Dengan kata lain, model runtut waktu mencoba melihat apa yang terjadi pada suatu kurun waktu tertentu dan menggunakan data runtut waktu masa lalu untuk memprediksi.

Model Kausal

- Model kausal memasukkan dan menguji variabel-variabel yang diduga mempengaruhi variabel *dependen*.
- Model kausal biasanya menggunakan analisis regresi untuk menentukan mana variabel yang signifikan mempengaruhi variabel *dependen*
- Model kausal juga dapat menggunakan metode ARIMA atau Box-Jenkins untuk mencari model terbaik yang dapat digunakan dalam peramalan

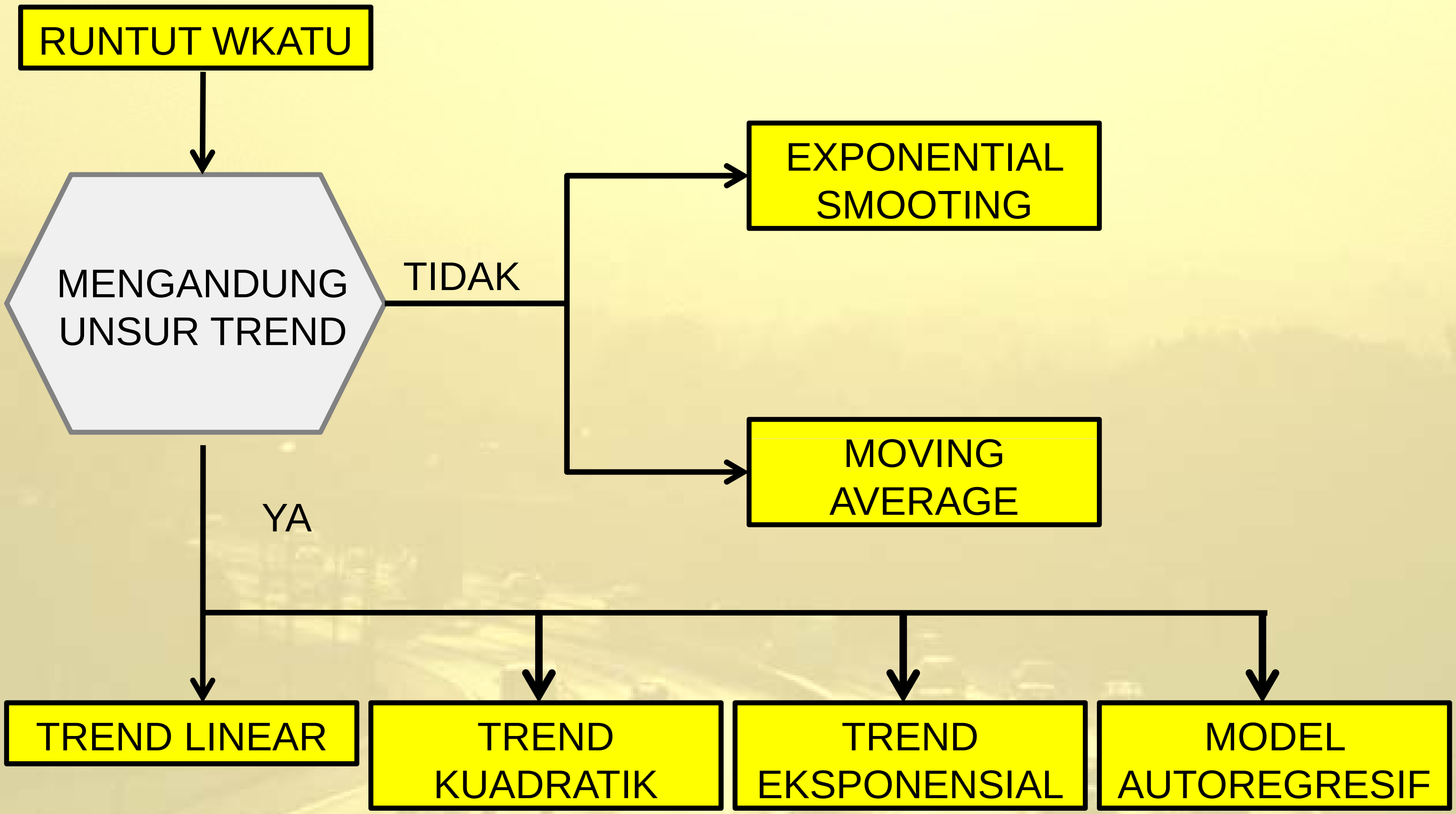
Model Kualitatif

- model kualitatif berupaya memasukkan faktor-faktor subyektif dalam model peramalan
- Model semacam ini diharapkan akan sangat bermanfaat apabila data kuantitatif yang akurat sulit diperoleh

Model Peramalan Runtut Waktu

- Model runtut waktu yang dipilih untuk peramalan tergantung dari apakah data yang digunakan mengandung unsur *trend* atau tidak
- Apabila data tidak mengandung unsur trend, maka teknik peramalan yang dapat digunakan adalah dengan ***penghalusan eksponensial (expionential smoothing),*** dan ***rata-rata bergerak (moving average)***
- Apabila data runtut waktu mengandung unsur trend, maka peramalan yang dapat digunakan adalah teknik trend linear, trend kuadratik, trend eksponensial, artau model autoregresif.
- Bagaimana mengidentifikasi apakah suatu data runtut waktu mengandung komponen trend atau tidak? Salah satu cara yang bisa dilakukan adalah dengan menggunakan **Uji Akar Unit**

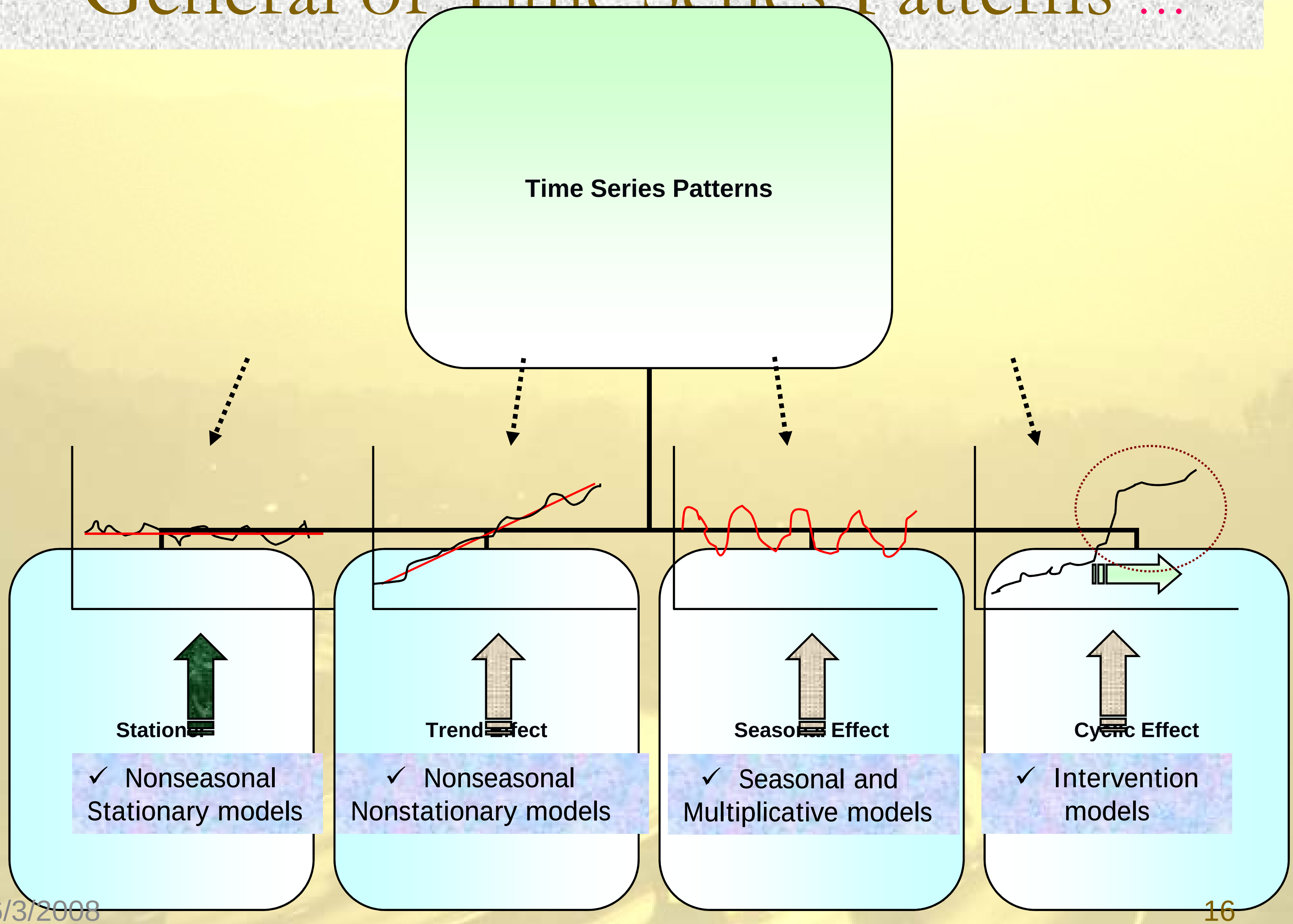
MODEL PERAMALAN RUNTUT WAKTU DENGAN ATAU TANPA TREN



Empat komponen yang ditemukan dalam analisis runtut waktu adalah:

1. ***Trend,*** **yaitu komponen jangka panjang yang mendasari pertumbuhan (atau penurunan) suatu data runtut waktu.**
2. ***Siklikal (cyclical),*** **yaitu suatu pola fluktuasi atau siklus dari data runtut waktu akibat perubahan kondisi ekonomi.**
3. ***Musiman (seasonal),*** **yaitu fluktuasi musiman yang sering dijumpai pada data kuartalan, bulanan atau mingguan.**
4. ***Tak beraturan (irregular),*** **yaitu pola acak yang disebabkan oleh peristiwa yang tidak dapat diprediksi atau tidak beraturan, seperti perang, pemogokan, pemilu, atau longsor maupun bencana alam lainnya**

General of Time Series Patterns …



Time Series Analysis

Deret berkala adalah suatu pengamatan atas suatu kumpulan variabel kuantitatif dari waktu ke waktu.

Contoh

- angka indeks rata-rata industri Dow Jones
- data historis penjualan, persediaan, jumlah pelanggan, tingkat bunga, biaya-biaya, dan lain-lain

Dunia Bisnis sangat tertarik akan peramalan dengan mengunakan variabel berkala

Sering, bahwa variabel independen adalah tidak tersedia untuk membangun model regresi dari variabel deret berkala

Dalam analisis deret berkala, kita meneliti perilaku dari suatu variabel masa lalu dalam rangka meramalkan perilakunya.masa depan

Pola data

General Time Series "PATTERN"

- Stationer
- Trend (linear or nonlinear)
- Seasonal (additive or multiplicative)
- Cyclic
- Calendar Variation

Pendekatan Analisis Deret Berkala

Ada banyak teknik deret berkala.
Ini biasanya mungkin untuk mengetahui teknik mana yang terbaik untuk data tertentu.
Biasanya mencoba beberapa teknik berbeda dan memilih salah satu terbaik.
Untuk menjadi suatu model deret berkala yang efektif, ini harus menyediakan beberapa teknikderet berkala di dalam “ tool box.”

Measuring Accuracy

- We need a way to compare different time series techniques for a given data set.
- Four common techniques are the:
  - mean absolute deviation, $\text{MAD} = \sum_{i=1}^{n} \frac{\left| Y_i - \hat{Y}_i \right|}{n}$
  - mean absolute percent error, $\text{MAPE} = \frac{100}{n} \sum_{i=1}^{n} \frac{\left| Y_i - \hat{Y}_i \right|}{Y_i}$
  - the mean square error, $\text{MSE} = \sum_{i=1}^{n} \frac{\left(Y_i - \hat{Y}_i \right)^2}{n}$
  - root mean square error. $\text{RMSE} = \sqrt{\text{MSE}}$

We will focus on the MSE

Extrapolation Models

- Extrapolation models try to account for the past behavior of a time series variable in an effort to predict the future behavior of the variable.

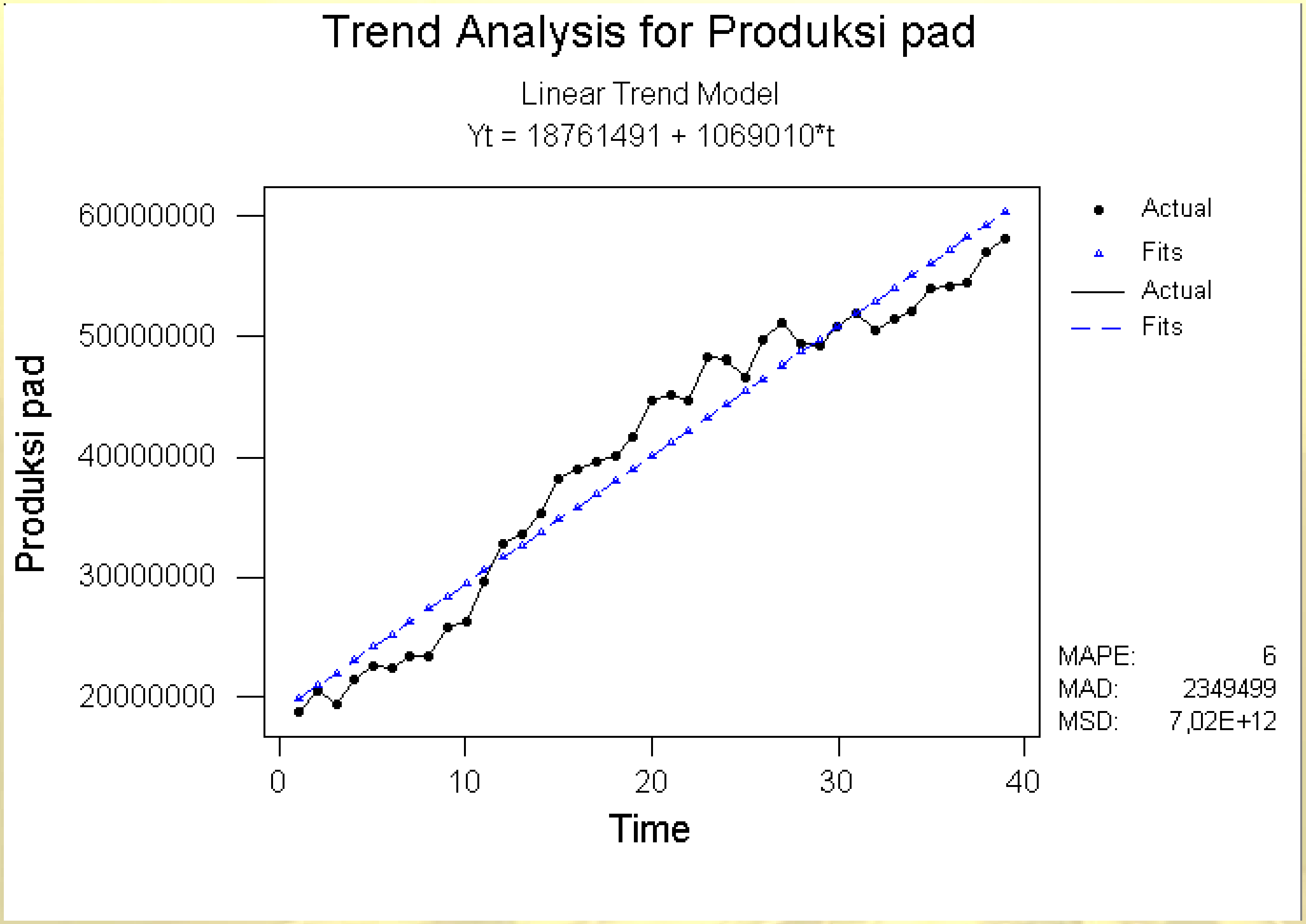
$$\hat{Y}_{t+1} = f\left(Y_t, Y_{t-1}, Y_{t-2}, \ldots\right)$$

- We'll first talk about several extrapolation techniques that are appropriate for stationary data.

An Example

- Hasil produksi padi Indonesia dari tahun 1970 sampai tahun 2008 sampai bulan Mei.
- Hasil produksi ini berdasarkan musiman
- Ada 39 tahun

tahun	produksi padi	tahun	produksi padi
1970	18.693.649	1990	45.178.751
1971	20.483.687	1991	44.688.247
1972	19.393.933	1992	48.240.009
1973	21.490.578	1993	48.181.087
1974	22.476.073	1994	46.641.524
1975	22.339.455	1995	49.744.140
1976	23.300.939	1996	51.101.506
1977	23.347.132	1997	49.377.054
1978	25.771.570	1998	49.236.692
1979	26.282.663	1999	50.866.387
1980	29.651.905	2000	51.898.852
1981	32.774.176	2001	50.460.782
1982	33.583.677	2002	51.489.694
1983	35.303.106	2003	52.137.604
1984	38.136.446	2004	54.088.468
1985	39.032.945	2005	54.151.097
1986	39.726.761	2006	54.454.937
1987	40.078.195	2007	57.051.679
1988	41.676.170	2008	58.268.796
1989	44.725.582		

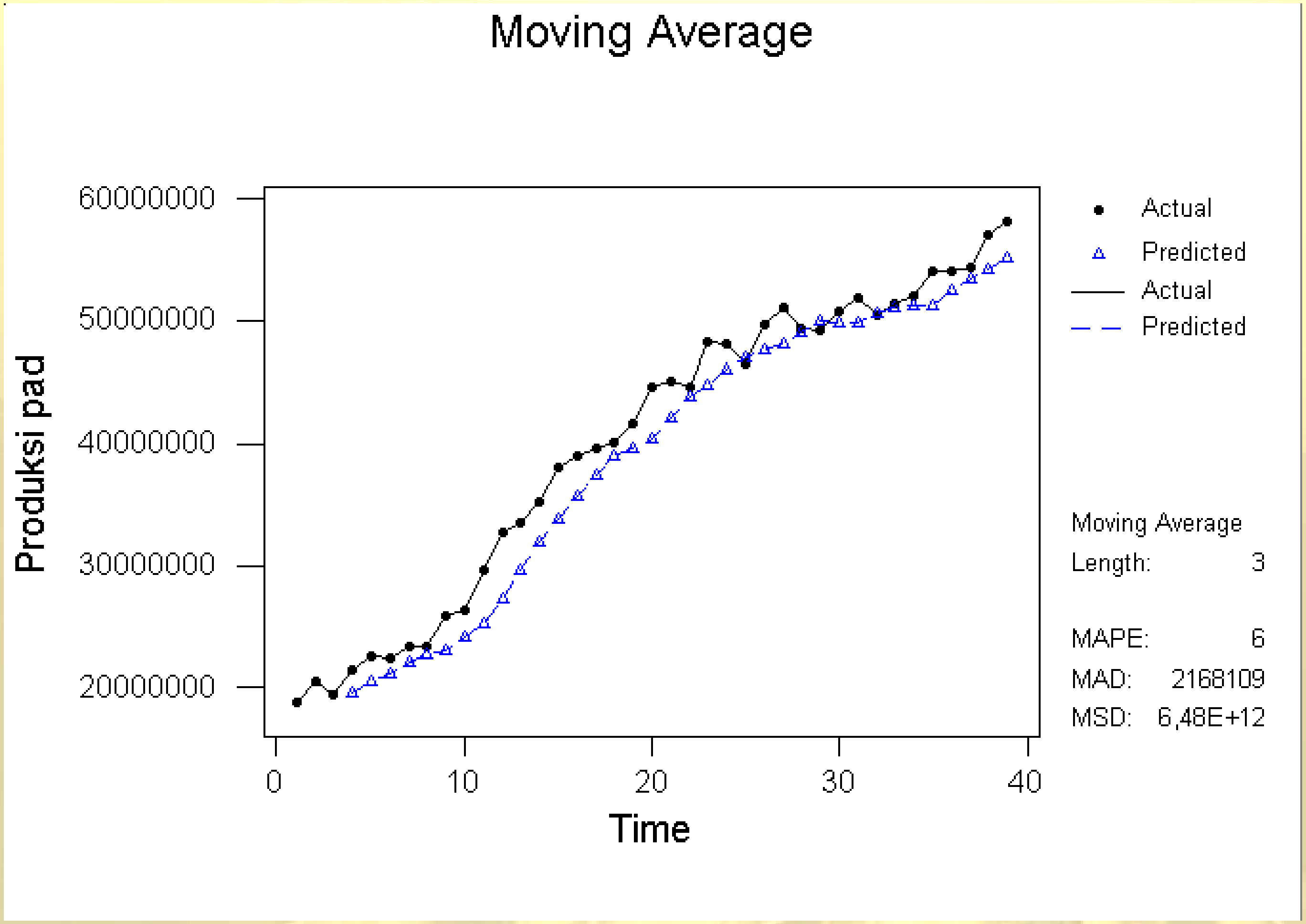

Trend Analysis for Produksi pad
Linear Trend Model
Yt = 18761491 + 1069010*t
Produksi pad
60000000
50000000
40000000
30000000
20000000
0
10
20
30
40
Time
Actual
Fits
Actual
Fits
MAPE: 6
MAD: 2349499
MSD: 7,02E+12

Moving Averages

$$\hat{Y}_{t+1} = \frac{Y_t + Y_{t-1} + Y_{t-k+1}}{k}$$

- No general method exists for determining k.
- We must try out several k values to see what works best.

Implementing the Model



A Comment on Comparing MSE Values

- Care should be taken when comparing MSE values of two different forecasting techniques.
- The lowest MSE may result from a technique that fits older values very well but fits recent values poorly.
- It is sometimes wise to compute the MSE using only the most recent values.

Forecasting With The Moving Average Model

Forecasts for time periods 25 and 26 at time period 24:

$$\hat{Y}_{25} = \frac{Y_{24} + Y_{23}}{2} = \frac{36 + 35}{2} = 35.5$$

$$\hat{Y}_{26} = \frac{\hat{Y}_{25} + Y_{24}}{2} = \frac{35.5 + 36}{2} = 35.75$$

Weighted Moving Average

- The moving average technique assigns equal weight to all previous observations

$$\hat{Y}_{t+1} = \frac{1}{k}Y_t + \frac{1}{k}Y_{t-1} + \cdots + \frac{1}{k}Y_{t-k-1}$$

- The weighted moving average technique allows for different weights to be assigned to previous observations.

$$\hat{Y}_{t+1} = w_1 Y_t + w_2 Y_{t-1} + \cdots + w_k Y_{t-k-1}$$

$$\text{where } 0 \le w_i \le 1 \text{ and } \sum w_i = 1$$

- We must determine values for k and the w_i

Forecasting With The Weighted Moving Average Model

Forecasts for time periods 25 and 26 at time period 24:

$$\hat{Y}_{25} = w_1 Y_{24} + w_2 Y_{23} = 0.291 \times 36 + 0.709 \times 35 = 35.29$$

$$\hat{Y}_{26} = w_1 \hat{Y}_{25} + w_2 Y_{24} = 0.291 \times 35.29 + 0.709 \times 36 = 35.79$$

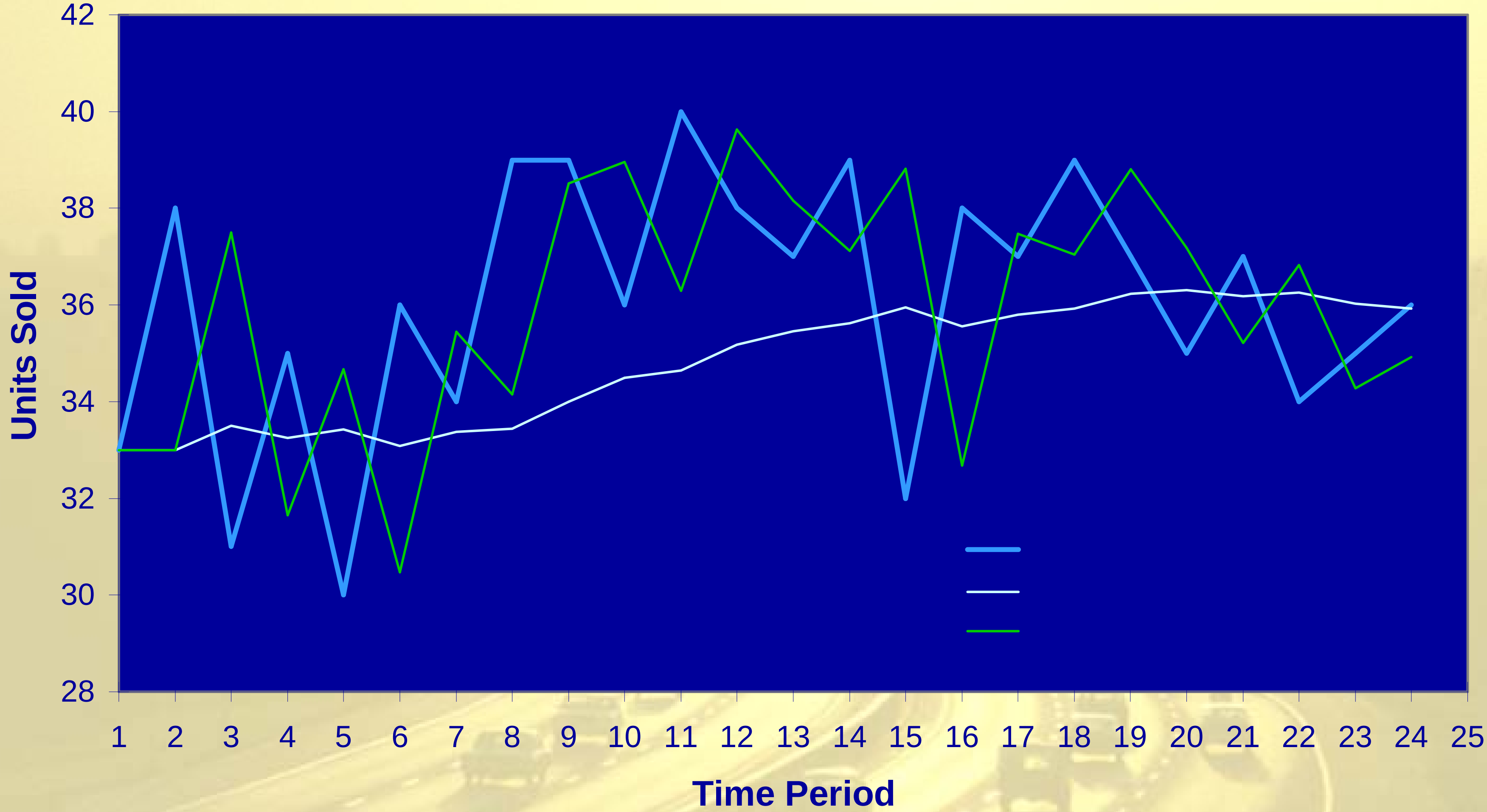
Exponential Smoothing

$$\hat{Y}_{t+1} = \hat{Y}_t + \alpha(Y_t - \hat{Y}_t)$$
$$\text{where } 0 \le \alpha \le 1$$

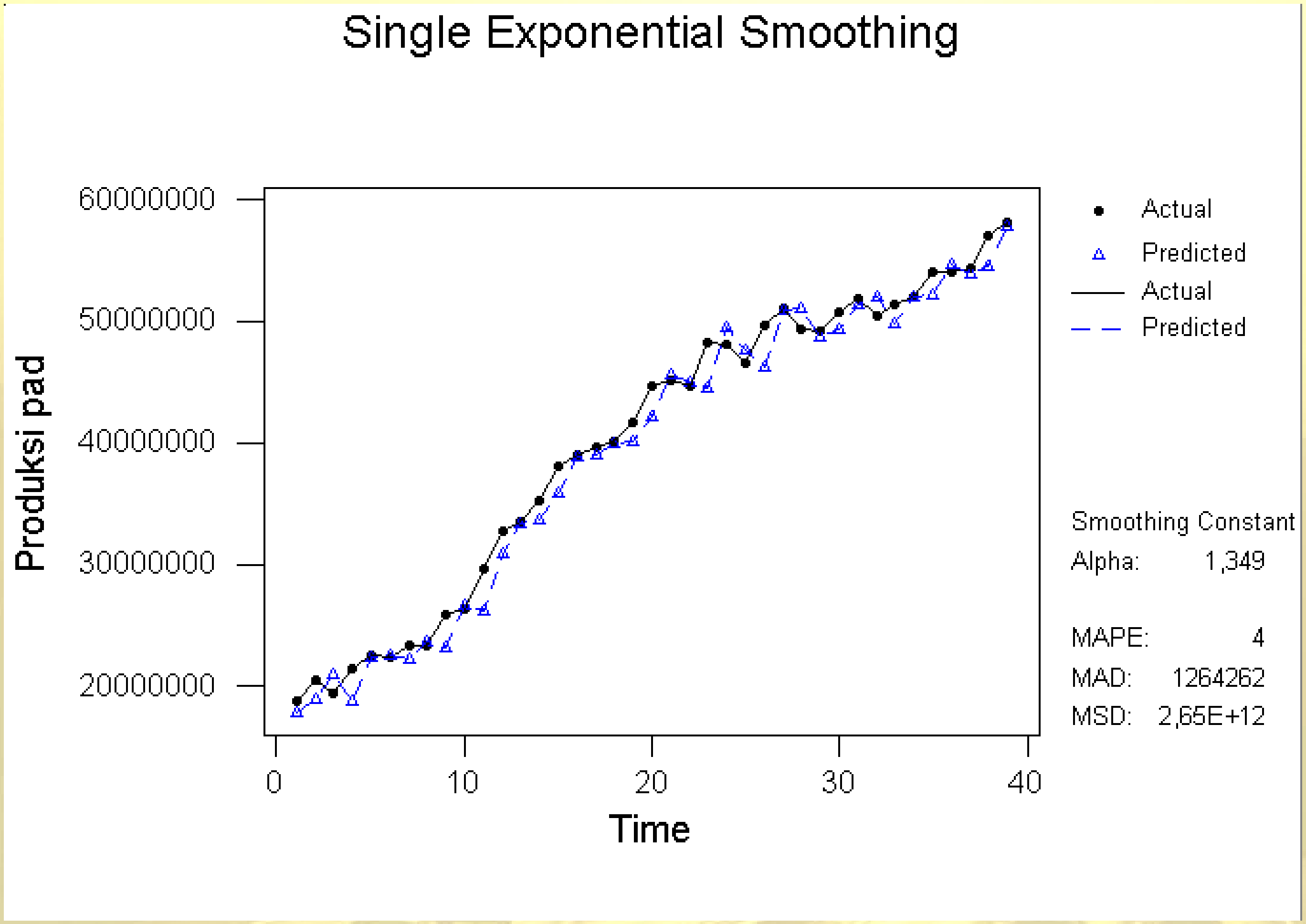
- It can be shown that the above equation is equivalent to:

$$\hat{Y}_{t+1} = \alpha Y_t + \alpha(1-\alpha)Y_{t-1} + \alpha(1-\alpha)^2 Y_{t-2} + \cdots + \alpha(1-\alpha)^n Y_{t-n} + \cdots$$

Examples of Two Exponential Smoothing Functions



Implementing the Model



Forecasting With The Exponential Smoothing Model

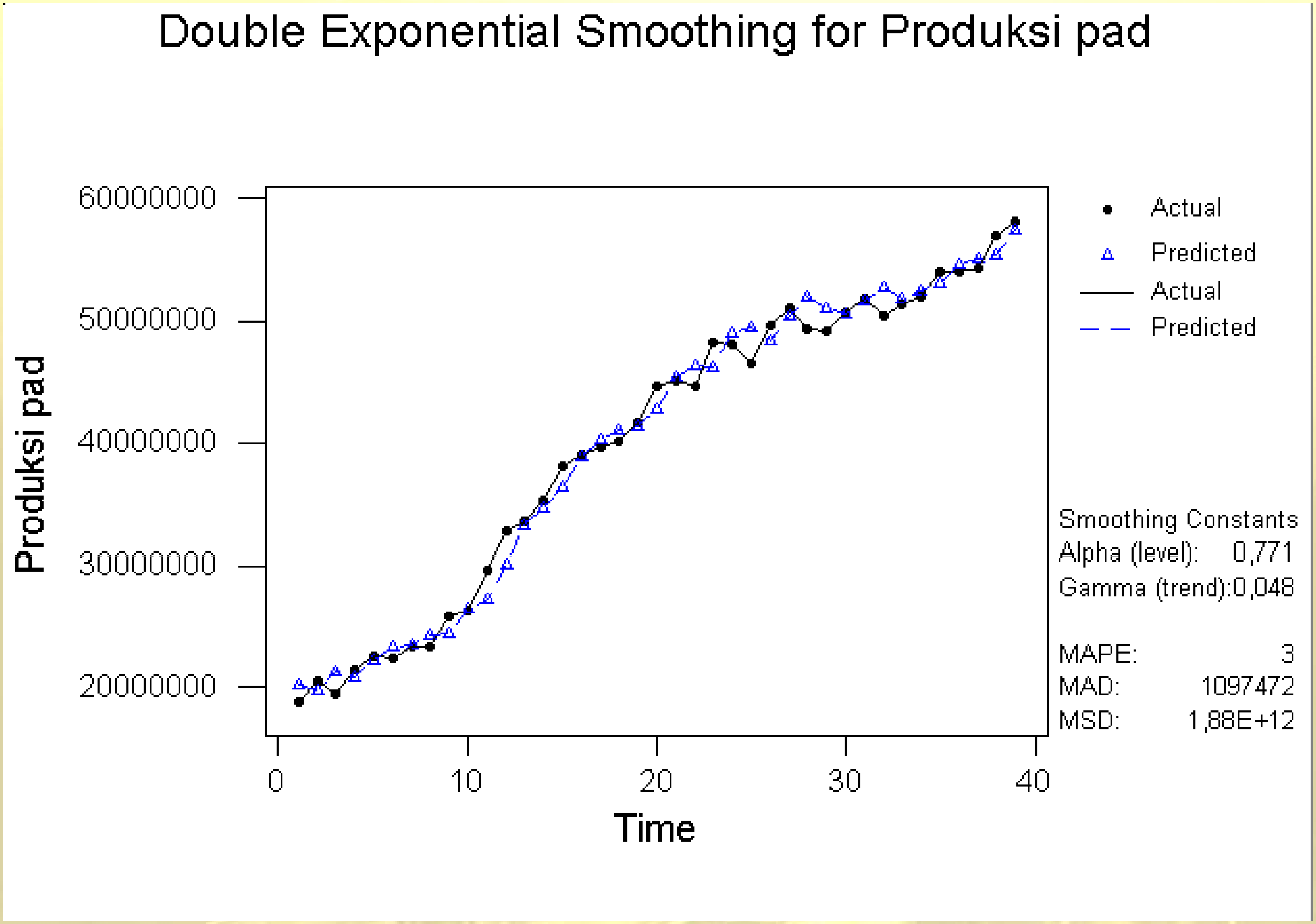
Forecasts for time periods 25 and 26 at time period 24:

$$\hat{Y}_{25} = \hat{Y}_{24} + \alpha(Y_{24} - \hat{Y}_{24}) = 35.74 + 0.268(36 - 35.74) = 35.81$$

$$\hat{Y}_{26} = \hat{Y}_{25} + \alpha(Y_{25} - \hat{Y}_{25}) \approx \hat{Y}_{25} + \alpha(\hat{Y}_{25} - \hat{Y}_{25}) = \hat{Y}_{25} = 35.81$$

Note that,

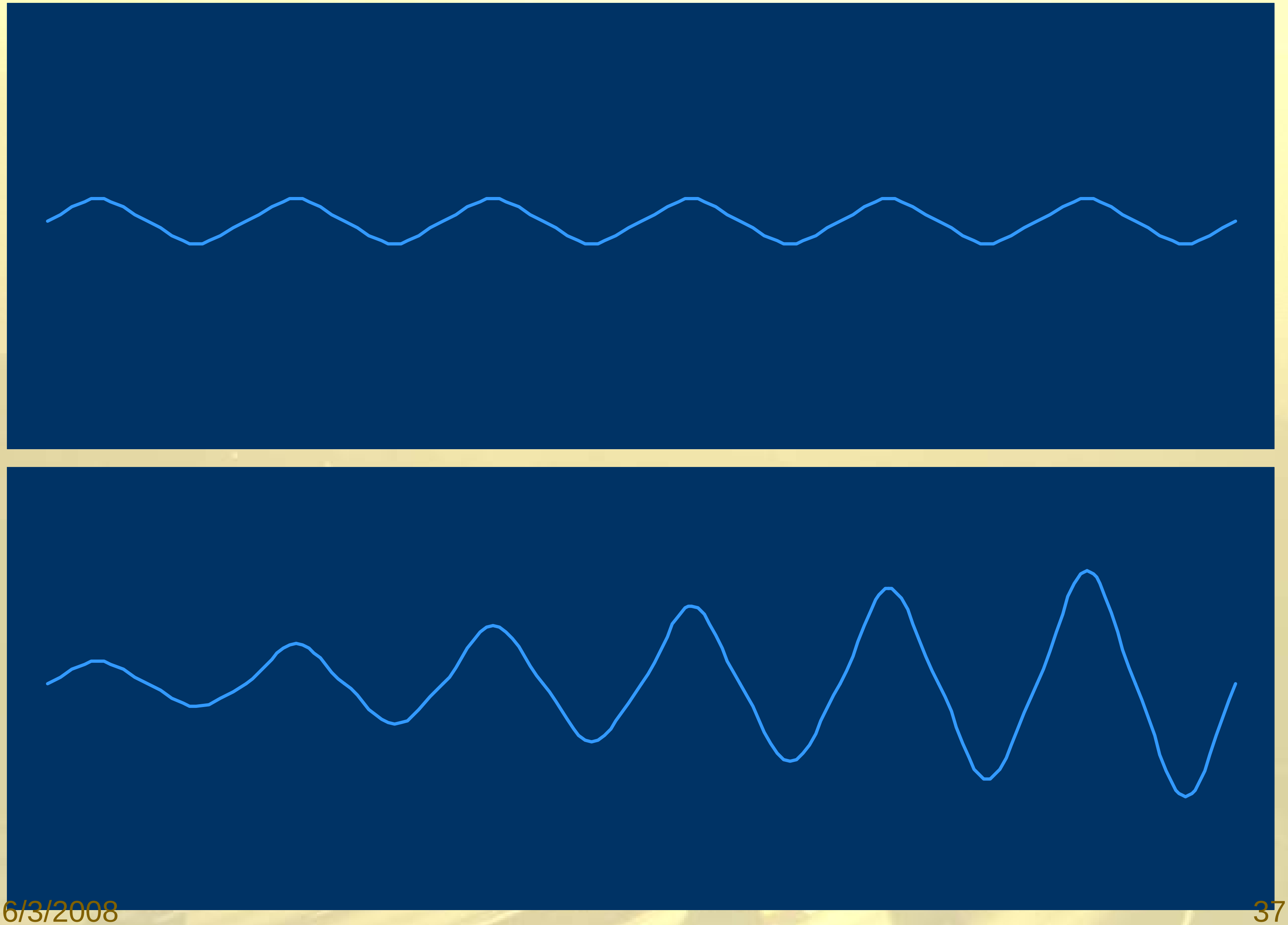
$$\hat{Y}_t = 35.81, \text{ for } t = 25,\ 26,\ 27,\ \ldots$$


Double Exponential Smoothing for Produksi pad
60000000
50000000
40000000
30000000
20000000
Produksi pad
0
10
20
30
40
Time
Actual
Predicted
Actual
Predicted
Smoothing Constants
Alpha (level): 0,771
Gamma (trend):0,048
MAPE: 3
MAD: 1097472
MSD: 1,88E+12

Seasonality

- Seasonality is a regular, repeating pattern in time series data.
- May be additive or multiplicative in nature...

Stationary Seasonal Effects



Stationary Data With Additive Seasonal Effects

$$\hat{Y}_{t+n} = E_t + S_{t+n-p}$$

where

$$E_t = \alpha(Y_t - S_{t-p}) + (1-\alpha)E_{t-1}$$

$$S_t = \beta(Y_t - E_t) + (1-\beta)S_{t-p}$$

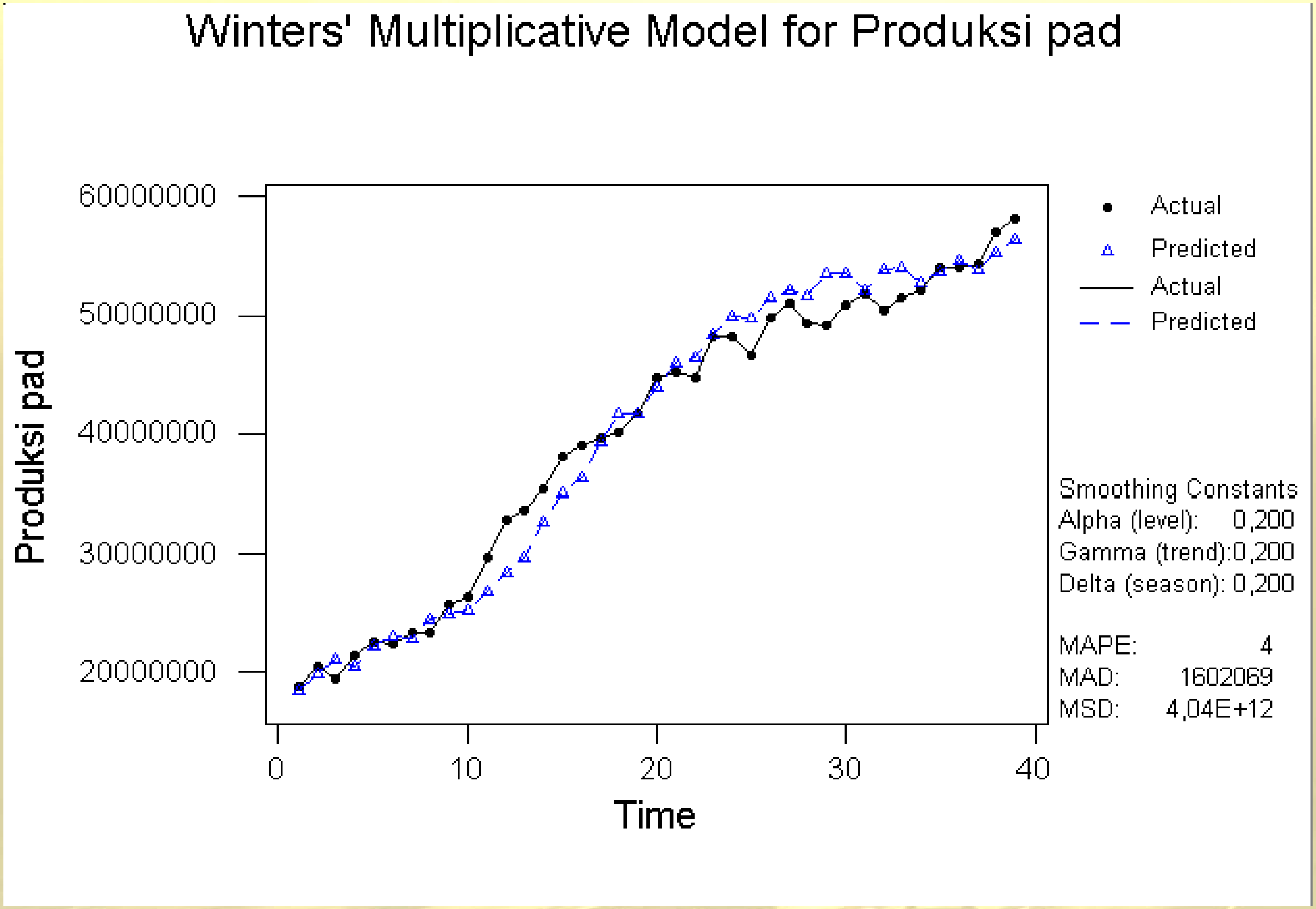
$$0 \le \alpha \le 1$$

$$0 \le \beta \le 1$$

p represents the number of seasonal periods

- E_t is the expected level at time period *t*.
- S_t is the seasonal factor for time period *t*.

Implementing the Model



Forecasting With The Additive Seasonal Effects Model

Forecasts for time periods 25 to 28 at time period 24:

$$\hat{Y}_{24+n} = E_{24} + S_{24+n-4}$$

$$\hat{Y}_{25} = E_{24} + S_{21} = 354.44 + 8.45 = 363.00$$

$$\hat{Y}_{26} = E_{24} + S_{22} = 354.44 - 17.82 = 336.73$$

$$\hat{Y}_{27} = E_{24} + S_{23} = 354.44 + 46.58 = 401.13$$

$$\hat{Y}_{28} = E_{24} + S_{24} = 354.44 - 31.73 = 322.81$$

Stationary Data With Multiplicative Seasonal Effects

$$\hat{Y}_{t+n} = E_t \times S_{t+n-p}$$

where

$$E_t = \alpha(Y_t/S_{t-p}) + (1-\alpha)E_{t-1}$$

$$S_t = \beta(Y_t/E_t) + (1-\beta)S_{t-p}$$

$$0 \le \alpha \le 1$$

$$0 \le \beta \le 1$$

p represents the number of seasonal periods

- E_t is the expected level at time period t.
- S_t is the seasonal factor for time period t.

Forecasting With The Multiplicative Seasonal Effects Model

Forecasts for time periods 25 to 28 at time period 24:

$$\hat{Y}_{24+n} = E_{24} \times S_{24+n-4}$$

$$\hat{Y}_{25} = E_{24} \times S_{21} = 353.95 \times 1.015 = 359.13$$

$$\hat{Y}_{26} = E_{24} \times S_{22} = 354.44 \times 0.946 = 334.94$$

$$\hat{Y}_{27} = E_{24} \times S_{23} = 354.44 \times 1.133 = 400.99$$

$$\hat{Y}_{28} = E_{24} \times S_{24} = 354.44 \times 0.912 = 322.95$$

Trend Models

- Trend is the long-term sweep or general direction of movement in a time series.
- We'll now consider some nonstationary time series techniques that are appropriate for data exhibiting upward or downward trends.

An Example

- WaterCraft Inc. is a manufacturer of personal water crafts (also known as jet skis).
- The company has enjoyed a fairly steady growth in sales of its products.
- The officers of the company are preparing sales and manufacturing plans for the coming year.
- Forecasts are needed of the level of sales that the company expects to achieve each quarter.
- See file

Double Moving Average

$$\hat{Y}_{t+n} = E_t + nT_t$$

where

$$E_t = 2M_t - D_t$$

$$T_t = 2(M_t - D_t)/(k-1)$$

$$M_t = (Y_t + Y_{t-1} + \cdots + Y_{t-k+1})/k$$

$$D_t = (M_t + M_{t-1} + \cdots + M_{t-k+1})/k$$

- E_t is the expected base level at time period *t*.
- T_t is the expected trend at time period *t*.

Forecasting With The Double Moving Average Model

Forecasts for time periods 21 to 24 at time period 20:

$$\hat{Y}_{20+n} = E_{20} + nT_{20}$$

$$\hat{Y}_{21} = E_{20} + 1T_{20} = 2385.33 + 1 \times 139.9 = 2525.23$$

$$\hat{Y}_{22} = E_{20} + 2T_{20} = 2385.33 + 2 \times 139.9 = 2665.13$$

$$\hat{Y}_{23} = E_{20} + 3T_{20} = 2385.33 + 3 \times 139.9 = 2805.03$$

$$\hat{Y}_{24} = E_{20} + 4T_{20} = 2385.33 + 4 \times 139.9 = 2944.94$$

Double Exponential Smoothing (Holt's Method)

$$\hat{Y}_{t+n} = E_t + nT_t$$

where

$$E_t = \alpha Y_t + (1-\alpha)(E_{t-1} + T_{t-1})$$

$$T_t = \beta(E_t - E_{t-1}) + (1-\beta)\, T_{t-1}$$

$$0 \le \alpha \le 1 \text{ and } 0 \le \beta \le 1$$

- E_t is the expected base level at time period t.
- T_t is the expected trend at time period t.

Forecasting With Holt's Model

Forecasts for time periods 21 to 24 at time period 20:

$$\hat{Y}_{20+n} = E_{20} + nT_{20}$$

$$\hat{Y}_{21} = E_{20} + 1T_{20} = 2336.8 + 1 \times 152.1 = 2488.9$$
$$\hat{Y}_{22} = E_{20} + 2T_{20} = 2336.8 + 2 \times 152.1 = 2641.0$$
$$\hat{Y}_{23} = E_{20} + 3T_{20} = 2336.8 + 3 \times 152.1 = 2793.1$$
$$\hat{Y}_{24} = E_{20} + 4T_{20} = 2336.8 + 4 \times 152.1 = 2945.2$$

Holt-Winter's Method For Additive Seasonal Effects

$$\hat{Y}_{t+n} = E_t + nT_t + S_{t+n-p}$$

where

$$E_t = \alpha\left(Y_t - S_{t-p}\right) + (1\text{-}\alpha)(E_{t-1} + T_{t-1})$$

$$T_t = \beta\left(E_t - E_{t-1}\right) + (1\text{-}\beta)T_{t-1}$$

$$S_t = \gamma\left(Y_t - E_t\right) + (1\text{-}\gamma)S_{t-p}$$

$$0 \le \alpha \le 1$$

$$0 \le \beta \le 1$$

$$0 \le \gamma \le 1$$

Forecasting With Holt-Winter's Additive Seasonal Effects Method

Forecasts for time periods 21 to 24 at time period 20:

$$\hat{Y}_{20+n} = E_{20} + nT_{20} + S_{20+n-4}$$

$$\hat{Y}_{21} = E_{20} + 1 \times T_{20} + S_{17} = 2253.3 + 1 \times 154.3 + 262.66 = 2670.3$$

$$\hat{Y}_{22} = E_{20} + 2 \times T_{20} + S_{18} = 2253.3 + 2 \times 154.3 - 312.59 = 2249.3$$

$$\hat{Y}_{23} = E_{20} + 3 \times T_{20} + S_{19} = 2253.3 + 3 \times 154.3 + 205.40 = 2921.6$$

$$\hat{Y}_{24} = E_{20} + 4 \times T_{20} + S_{20} = 2253.3 + 4 \times 154.3 + 386.12 = 3256.6$$

Holt-Winter's Method For Multiplicative Seasonal Effects

$$\hat{Y}_{t+n} = (E_t + nT_t)S_{t+n-p}$$

where

$$E_t = \alpha(Y_t / S_{t-p}) + (1-\alpha)(E_{t-1} + T_{t-1})$$

$$T_t = \beta(E_t - E_{t-1}) + (1-\beta)T_{t-1}$$

$$S_t = \gamma(Y_t / E_t) + (1-\gamma)S_{t-p}$$

$$0 \le \alpha \le 1$$

$$0 \le \beta \le 1$$

$$0 \le \gamma \le 1$$

Forecasting With Holt-Winter's Multiplicative Seasonal Effects Method

Forecasts for time periods 21 to 24 at time period 20:

$$\hat{Y}_{20+n} = \left(E_{20} + nT_{20}\right)S_{20+n-4}$$

$$\hat{Y}_{21} = (E_{20} + 1T_{20})S_{17} = (2217.6 + 1\times 137.3)1.152 = 2713.7$$

$$\hat{Y}_{22} = (E_{20} + 2T_{20})S_{18} = (2217.6 + 2\times 137.3)0.849 = 2114.9$$

$$\hat{Y}_{23} = (E_{20} + 3T_{20})S_{19} = (2217.6 + 3\times 137.3)1.103 = 2900.5$$

$$\hat{Y}_{24} = (E_{20} + 4T_{20})S_{20} = (2217.6 + 4\times 137.3)1.190 = 3293.9$$

The Linear Trend Model

$$\hat{Y}_t = b_0 + b_1 X_{1_t}$$

$$\text{where } X_{1_t} = t$$

For example:

$$X_{1_1} = 1,\ X_{1_2} = 2,\ X_{1_3} = 3,\ \ldots$$

Forecasting With The Linear Trend Model

Forecasts for time periods 21 to 24 at time period 20:

$$\hat{Y}_{21} = b_0 + b_1 X_{1_{21}} = 375.1 + 92.6255 \times 21 = 2320.3$$

$$\hat{Y}_{22} = b_0 + b_1 X_{1_{22}} = 375.1 + 92.6255 \times 22 = 2412.9$$

$$\hat{Y}_{23} = b_0 + b_1 X_{1_{23}} = 375.1 + 92.6255 \times 23 = 2505.6$$

$$\hat{Y}_{24} = b_0 + b_1 X_{1_{24}} = 375.1 + 92.6255 \times 24 = 2598.2$$

The TREND() Function

TREND(Y-range, X-range, X-value for prediction)

where:

Y-range is the spreadsheet range containing the dependent Y variable,

X-range is the spreadsheet range containing the independent X variable(s),

X-value for prediction is a cell (or cells) containing the values for the independent X variable(s) for which we want an estimated value of Y.

Note: The TREND( ) function is dynamically updated whenever any inputs to the function change. However, it does not provide the statistical information provided by the regression tool. It is best two use these two different approaches to doing regression in conjunction with one another.

The Quadratic Trend Model

$$\hat{Y}_t = b_0 + b_1 X_{1_t} + b_2 X_{2_t}$$

where $X_{1_t} = t$ and $X_{2_t} = t^2$

Forecasting With The Quadratic Trend Model

Forecasts for time periods 21 to 24 at time period 20:

$$\hat{Y}_{21} = b_0 + b_1 X_{1_{21}} + b_2 X_{2_{21}} = 653.67 + 16.671 \times 21 + 3.617 \times 21^2 = 2598.9$$

$$\hat{Y}_{22} = b_0 + b_1 X_{1_{22}} + b_2 X_{2_{22}} = 653.67 + 16.671 \times 22 + 3.617 \times 22^2 = 2771.1$$

$$\hat{Y}_{23} = b_0 + b_1 X_{1_{23}} + b_2 X_{2_{23}} = 653.67 + 16.671 \times 23 + 3.617 \times 23^2 = 2950.4$$

$$\hat{Y}_{24} = b_0 + b_1 X_{1_{24}} + b_2 X_{2_{24}} = 653.67 + 16.671 \times 24 + 3.617 \times 24^2 = 3137.1$$

Computing Multiplicative Seasonal Indices

- We can compute multiplicative seasonal adjustment indices for period p as follows:

$$S_p = \frac{\sum_i \frac{Y_i}{\hat{Y}_i}}{n_p}, \text{ for all } i \text{ occuring in season } p$$

- The final forecast for period i is then

$$\hat{Y}_i \text{ adjusted} = \hat{Y}_i \times S_p, \text{ for any } i \text{ occuring in season } p$$

Forecasting With Seasonal Factors Applied To The Quadratic Trend Model

Forecasts for time periods 21 to 24 at time period 20:

$$\hat{Y}_{21} = (b_0 + b_1 X_{1_{21}} + b_2 X_{2_{21}}) S_1 = 2598.9 \times 105.7\% = 2747.8$$

$$\hat{Y}_{22} = (b_0 + b_1 X_{1_{22}} + b_2 X_{2_{22}}) S_2 = 2771.1 \times 80.1\% = 2219.6$$

$$\hat{Y}_{23} = (b_0 + b_1 X_{1_{23}} + b_2 X_{2_{23}}) S_3 = 2950.5 \times 103.1\% = 3041.4$$

$$\hat{Y}_{24} = (b_0 + b_1 X_{1_{24}} + b_2 X_{2_{24}}) S_4 = 3137.2 \times 111.1\% = 3486.1$$

Summary of the Calculation and Use of Seasonal Indices

1. Create a trend model and calculate the estimated value ($\hat{Y}_t$) for each observation in the sample.
2. For each observation, calculate the ratio of the actual value to the predicted trend value: $Y_t / \hat{Y}_t$.

   (For additive effects, compute the difference: $Y_t - \hat{Y}_t$).
3. For each season, compute the average of the ratios calculated in step 2. These are the seasonal indices.
4. Multiply any forecast produced by the trend model by the appropriate seasonal index calculated in step 3. (For additive seasonal effects, add the appropriate factor to the forecast.)

Refining the Seasonal Indices

- Note that Solver can be used to *simultaneously* determine the optimal values of the seasonal indices *and* the parameters of the trend model being used.
- There is no guarantee that this will produce a better forecast, but it should produce a model that fits the data better in terms of the MSE.

See file Fig11-39.xls

Seasonal Regression Models

- Indicator variables may also be used in regression models to represent seasonal effects.
- If there are *p* seasons, we need *p* -1 indicator variables.

- Our example problem involves quarterly data, so *p*=4 and we define the following 3 indicator variables:

$$X_{3_t} = \begin{cases} 1, & \text{if } Y_t \text{ is an observation from quarter 1} \\ 0, & \text{otherwise} \end{cases}$$

$$X_{4_t} = \begin{cases} 1, & \text{if } Y_t \text{ is an observation from quarter 2} \\ 0, & \text{otherwise} \end{cases}$$

$$X_{5_t} = \begin{cases} 1, & \text{if } Y_t \text{ is an observation from quarter 3} \\ 0, & \text{otherwise} \end{cases}$$

Implementing the Model

- The regression function is:

$$\hat{Y}_t = b_0 + b_1 X_{1_t} + b_2 X_{2_t} + b_3 X_{3_t} + b_4 X_{4_t} + b_5 X_{5_t}$$

$$\text{where } X_{1_t} = t \text{ and } X_{2_t} = t^2$$

See file Fig11-42.xls

Forecasting With The Seasonal Regression Model

Forecasts for time periods 21 to 24 at time period 20:

$$\hat{Y}_{21} = 824.471 + 17.319(21) + 3.485(21)^2 - 86.805(1) - 424.736(0) - 123.453(0) = 2638.5$$

$$\hat{Y}_{22} = 824.471 + 17.319(22) + 3.485(22)^2 - 86.805(0) - 424.736(1) - 123.453(0) = 2467.7$$

$$\hat{Y}_{23} = 824.471 + 17.319(23) + 3.485(23)^2 - 86.805(0) - 424.736(0) - 123.453(1) = 2943.2$$

$$\hat{Y}_{24} = 824.471 + 17.319(24) + 3.485(24)^2 - 86.805(0) - 424.736(0) - 123.453(0) = 3247.8$$

StatTools

- StatTools is an add-in that simplifies the process of performing time series analysis in Excel.
- A trial version of StatTools is available on the CD-ROM accompanying this book.
- For more information on StatTools see:

  http://www.palisade.com

Combining Forecasts

- It is also possible to combine forecasts to create a composite forecast.
- Suppose we used three different forecasting methods on a given data set.

- Denote the predicted value of time period t using each method as follows:

$$F_{1_t}, F_{2_t}, \text{and } F_{3_t}$$

- We could create a composite forecast as follows:

$$\hat{Y}_t = b_0 + b_1 F_{1_t} + b_2 F_{2_t} + b_3 F_{3_t}$$

ARIMA Model: Produksi padi

ARIMA model for Produksi padi

Estimates at each iteration

Iteration	SSE	Parameters			
0	451374984309793	0,100	0,100	0,100	0,100
1	368364447306473	0,021	0,199	0,250	0,152
2	325456920491419	-0,129	0,154	0,348	0,186
3	285571989081979	-0,279	0,118	0,475	0,229
4	245711224167492	-0,324	0,193	0,542	0,187
5	187899115792845	-0,474	0,260	0,657	0,041
6	155112700780088	-0,527	0,353	0,704	-0,109
7	141714161123773	-0,447	0,503	0,647	-0,201
8	129630495049884	-0,368	0,653	0,585	-0,287
9	117486920579732	-0,301	0,803	0,523	-0,376
10	101549593049237	-0,291	0,953	0,498	-0,503
11	87502739697009	-0,393	1,051	0,567	-0,653
12	81737678770148	-0,543	1,076	0,614	-0,708
13	79314052359269	-0,619	1,074	0,596	-0,687
14	79233477547537	-0,622	1,073	0,587	-0,677
15	79162421478230	-0,620	1,072	0,581	-0,671
16	79124151604006	-0,619	1,071	0,577	-0,667
17	79104368397552	-0,619	1,071	0,576	-0,665
18	79092164140468	-0,619	1,070	0,575	-0,664
19	79085747120700	-0,620	1,070	0,575	-0,664

Relative change in each estimate less than 0,0010

Final Estimates of Parameters

Type	Coef	SE Coef	T	P
AR 1	-0,6197	0,1611	-3,85	0,001
MA 1	1,0698	0,0298	35,85	0,000
MA 2	0,5753	0,1894	3,04	0,005
MA 3	-0,6641	0,1882	-3,53	0,001

Differencing: 3 regular differences

Number of observations: Original series 39, after differencing 36

Residuals: SS = 70758139198041 (backforecasts excluded)

MS = 2211191849939 DF = 32

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	20,9	38,4	*	*
DF	8	20	*	*
P-Value	0,008	0,008	*	*

Model-model Time Series Regression

1. **Model Regresi untuk LINEAR TREND**

   $Y_t = a + b.t + \text{error}$ ⇨ **t = 1, 2, … (dummy waktu)**

2. **Model Regresi untuk Data SEASONAL (variasi konstan)**

   $Y_t = a + b_1 D_1 + \ldots + b_{S-1} D_{S-1} + \text{error}$

   dengan : **$D_1, D_2, \ldots, D_{S-1}$ adalah dummy waktu dalam satu periode seasonal.**

3. **Model Regresi untuk Data dengan LINEAR TREND dan SEASONAL (variasi konstan)**

   $Y_t = a + b.t + c_1 D_1 + \ldots + c_{S-1} D_{S-1} + \text{error}$

   ➲ **Gabungan model 1 dan 2.**

Naïve Model

- The **recent periods** are the best predictors of the future.

1. **The simplest model for stationary data is**

$$\Rightarrow \quad \hat{Y}_{t+1} = Y_t$$

2. **The simplest model for trend data is**

$$\Rightarrow \quad \hat{Y}_{t+1} = Y_t + (Y_t - Y_{t-1}) \quad \text{or}$$

$$\Rightarrow \quad \hat{Y}_{t+1} = Y_t \frac{Y_t}{Y_{t-1}}$$

3. **The simplest model for seasonal data is**

$$\Rightarrow \quad \hat{Y}_{t+1} = Y_{(t+1)-s}$$

Average Methods

1. **Simple Averages**
   - obtained by finding the **mean for <u>all the relevant values</u>** and then **using this mean to forecast the next period**.

$$\Rightarrow \quad \hat{Y}_{t+1} = \sum_{t=1}^{n} \frac{Y_t}{n}$$

for stationary data

2. **Moving Averages**
   - obtained by finding the **mean for <u>a specified set of values</u>** and then **using this mean to forecast the next period**.

$$\Rightarrow \quad M_t = \hat{Y}_{t+1} = \frac{(Y_t + Y_{t-1} + \ldots + Y_{t-n+1})}{n}$$

for stationary data

Average Methods … (continued)

3. Double Moving Averages

- one set of moving averages is computed, and then a second set is computed as a moving average of the first set.

(i). $M_t = \hat{Y}_{t+1} = \dfrac{(Y_t + Y_{t-1} + \ldots + Y_{t-n+1})}{n}$

(ii). $M'_t = \dfrac{(M_t + M_{t-1} + \ldots + M_{t-n+1})}{n}$

(iii). $a_t = 2M_t - M'_t$

(iv). $b_t = \dfrac{2}{n-1}(M_t - M'_t)$

$\Rightarrow \hat{Y}_{t+p} = a_t + b_t p$ **for a linear trend data**

Exponential Smoothing Methods

- ✓ **Single Exponential Smoothing** ⇨ for **stationary** data

  $$\hat{Y}_{t+1} = \alpha Y_t + (1-\alpha)\hat{Y}_t$$

- ✓ Exponential Smoothing Adjusted for **Trend** : **Holt's Method**

  1. The exponentially smoothed series :

     $$A_t = \alpha Y_t + (1-\alpha)(A_{t-1} + T_{t-1})$$

  2. The trend estimate :

     $$T_t = \beta (A_t - A_{t-1}) + (1-\beta) T_{t-1}$$

  3. Forecast **p** periods into the future :

     $$\hat{Y}_{t+p} = A_t + pT_t$$

Exponential Smoothing Adjusted for **Trend** and **Seasonal** Variation : **Winter's Method**

1. The exponentially **smoothed** series :

$$A_t = \alpha \frac{Y_t}{S_{t-L}} + (1-\alpha)(A_{t-1} + T_{t-1})$$

2. The **trend** estimate :

$$T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$$

3. The **seasonality** estimate :

$$S_t = \gamma \frac{Y_t}{A_t} + (1-\gamma)S_{t-1}$$

4. ***Forecast p*** periods into the future :

$$\hat{Y}_{t+p} = (A_t - pT_t)S_{t-L+p}$$

Three parameters models