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Engineering Economics and Project Management

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17.1 Engineering Economic Decisions

Decisions made during the engineering design phase of product development determine the majority (some say 85%) of the costs of manufacturing that product. Thus, a competent engineer in the 21st century must have an understanding of the principles of economics as well as engineering. This chapter examines the most important economic concepts that should be understood by engineers.

Engineers participate in a variety of decision-making processes, ranging from manufacturing to marketing to financing decisions. They must make decisions involving materials, plant facilities, the in-house capabilities of company personnel, and the effective use of capital assets such as buildings and machinery. One of the engineer's primary tasks is to plan for the acquisition of equipment (fixed asset) that will enable the firm to design and produce products economically. These decisions are called *engineering economic decisions*.

17.2 Establishing Economic Equivalence

A typical engineering economic decision involves two dissimilar types of dollar amounts. First, there is the investment, which is usually made in a lump sum at the beginning of the project, a time that for analytical purposes is called today, or time 0. Second, there is a stream of cash benefits that are expected to result from this investment over a period of future years.

In such a fixed asset investment funds are committed today in the expectation of earning a return in the future. In the case of a bank loan, the future return takes the form of interest plus repayment of the principal. This is known as the *loan cash flow*. In the case of the fixed asset, the future return takes the form of cash generated by productive use of the asset. The representation of these future earnings along with the capital expenditures and annual expenses (such as wages, raw materials, operating costs, maintenance costs, and income taxes) is the *project cash flow*. This similarity between the loan cash flow and the project cash flow brings us an important conclusion—that is, first we need to find a way to evaluate a money series occurring at different points in time. Second, if we understand how to evaluate a loan cash flow series, we can use the same concept to evaluate the project cash flow series.

Interest: The Cost of Money

Money left in a savings account earns interest so that the balance over time is greater than the sum of the deposits. In the financial world, money itself is a commodity, and like other goods that are bought and sold, money costs money. The cost of money is established and measured by an *interest rate*, a percentage that is periodically applied and added to an amount (or varying amounts) of money over a specified length of time. When money is borrowed, the interest paid is the charge to the borrower for the use of the lender's property; when money is loaned or invested, the interest earned is the lender's gain from providing a good to another. *Interest*, then, may be defined as the cost of having money available for use.

The operation of interest reflects the fact that money has a time value. This is why amounts of interest depend on lengths of time; interest rates, for example, are typically given in terms of a percentage per year. This principle of the time value of money can be formally defined as follows: the economic value of a sum depends on when it is received. Because money has earning power over time (it can be put to work, earning more money for its owner), a dollar received today has a greater value than a dollar received at some future time.

The changes in the value of a sum of money over time can become extremely significant when we deal with large amounts of money, long periods of time, or high interest rates. For example, at a current annual interest rate of 10%, \$1 million will earn \$100,000 in interest in a year; thus, waiting a year to receive \$1 million clearly involves a significant sacrifice. In deciding among alternative proposals, we must take into account the operation of interest and the time value of money to make valid comparisons of different amounts at various times.

The Elements of Transactions Involving Interest

Many types of transactions involve interest — for example, borrowing or investing money, purchasing machinery on credit — but certain elements are common to all of them:

1. Some initial amount of money, called the *principal* (P) in transactions of debt or investment
2. The *interest rate* (i), which measures the cost or price of money, expressed as a percentage per period of time
3. A period of time, called the *interest period* (or *compounding period*), that determines how frequently interest is calculated
4. The specified length of time that marks the duration of the transaction and thereby establishes a certain *number of interest periods* (N)
5. A *plan for receipts or disbursements* (A_n) that yields a particular cash flow pattern over the length of time (for example, we might have a series of equal monthly payments [A] that repay a loan)
6. A *future amount of money* (F) that results from the cumulative effects of the interest rate over a number of interest periods

Cash Flow Diagrams

It is convenient to represent problems involving the time value of money in graphic form with a cash flow diagram (see Figure 17.2.1), which represents time by a horizontal line marked off with the number of interest periods specified. The cash flows over time are represented by arrows at the relevant periods: upward arrows for positive flows (receipts) and downward arrows for negative flows (disbursements).

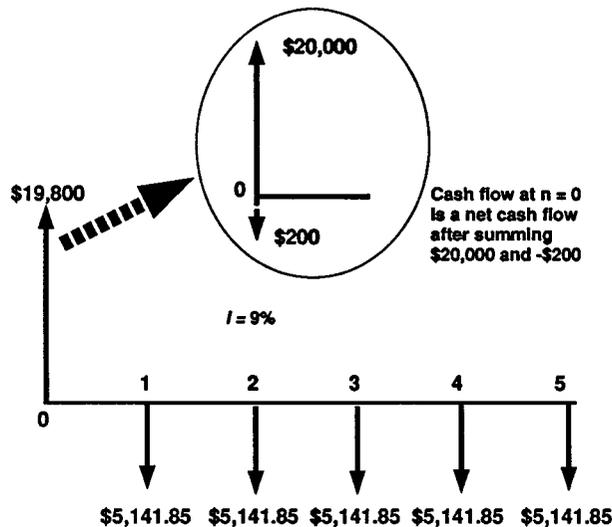


FIGURE 17.2.1 A cash flow diagram for a loan transaction — borrow \$20,000 now and pay off the loan with five equal annual installments of \$5,141.85. After paying \$200 for the loan origination fee, the net amount of financing is \$19,800. The borrowing interest rate is 9%.

End-of-Period Convention

In practice, cash flows can occur at the beginning or in the middle of an interest period, or at practically any point in time. One of the simplifying assumptions we make in engineering economic analysis is the *end-of-period convention*, which is the practice of placing all cash flow transactions at the end of an interest period. This assumption relieves us of the responsibility of dealing with the effects of interest within an interest period, which would greatly complicate our calculations.

Compound Interest

Under the compound interest scheme, the interest in each period is based on the total amount owed at the end of the previous period. This total amount includes the original principal plus the accumulated interest that has been left in the account. In this case, you are in effect increasing the deposit amount by the amount of interest earned. In general, if you deposited (invested) P dollars at interest rate i , you would have $P + iP = P(1 + i)$ dollars at the end of one period. With the entire amount (principal and interest) reinvested at the same rate i for another period, you would have, at the end of the second period,

$$\begin{aligned} P(1 + i) + i[P(1 + i)] &= P(1 + i)(1 + i) \\ &= P(1 + i)^2 \end{aligned}$$

This interest-earning process repeats, and after N periods, the total accumulated value (balance) F will grow to

$$F = P(1 + i)^N \quad (17.2.1)$$

Equivalence Calculations

Economic equivalence refers to the fact that a cash flow — whether it is a single payment or a series of payments — can be said to be converted to an *equivalent* cash flow at any point in time; thus, for any sequence of cash flows, we can find an equivalent single cash flow at a given interest rate and a given time.

Equivalence calculations can be viewed as an application of the compound interest relationships developed in Equation 17.2.1. The formula developed for calculating compound interest, $F = P(1 + i)^N$, expresses the equivalence between some present amount, P , and a future amount, F , for a given interest rate, i , and a number of interest periods, N . Therefore, at the end of a 3-year investment period at 8%, \$1000 will grow to

$$\$1000(1 + 0.08)^3 = \$1259.71$$

Thus at 8% interest, \$1000 received now is equivalent to \$1,259.71 received in 3 years and we could trade \$1000 now for the promise of receiving \$1259.71 in 3 years. Example 17.2.1 demonstrates the application of this basic technique.

Example 17.2.1 — Equivalence

Suppose you are offered the alternative of receiving either \$3000 at the end of 5 years or P dollars today. There is no question that the \$3000 will be paid in full (no risk). Having no current need for the money, you would deposit the P dollars in an account that pays 8% interest. What value of P would make you indifferent in your choice between P dollars today and the promise of \$3000 at the end of 5 years from now?

Discussion. Our job is to determine the present amount that is economically equivalent to \$3000 in 5 years, given the investment potential of 8% per year. Note that the problem statement assumes that you would exercise your option of using the earning power of your money by depositing it. The “indifference” ascribed to you refers to economic indifference; that is, within a marketplace where 8% is the applicable interest rate, you could trade one cash flow for the other.

Solution. From Equation (17.2.1), we establish

$$\$3000 = P(1 + 0.08)^5$$

Rearranging to solve for P,

$$P = \$3000 / (1 + 0.08)^5 = \$2042$$

Comments. In this example, it is clear that if P is anything less than \$2042, you would prefer the promise of \$3000 in 5 years to P dollars today; if P were greater than \$2042, you would prefer P. It is less obvious that at a lower interest rate, P must be higher to be equivalent to the future amount. For example, at $i = 4\%$, $P = \$2466$.

Interest Formulas

In this section is developed a series of interest formulas for use in more complex comparisons of cash flows. It classifies four major categories of cash flow transactions, develops interest formulas for them, and presents working examples of each type.

Single Cash Flow Formulas

We begin our coverage of interest formulas by considering the simplest cash flows: single cash flows. Given a present sum P invested for N interest periods at interest rate i, what sum will have accumulated at the end of the N periods? You probably noticed quickly that this description matches the case we first encountered in describing compound interest. To solve for F (the future sum) we use Equation (17.2.1):

$$F = P(1 + i)^N = P(F/P, i, N)$$

Because of its origin in compound interest calculation, the factor $(F/P, i, N)$, which is read as “find F, given P, i, and N” is known as the *single payment compound amount factor*. Like the concept of equivalence, this factor is one of the foundations of engineering economic analysis. Given this factor, all the other important interest formulas can be derived. This process of finding F is often called the compounding process. (Note the time-scale convention. The first period begins at $n = 0$ and ends at $n = 1$.) Thus, in the preceding example, where we had $F = \$1000(1.08)^3$, we can write $F = \$1000(F/P, 8\%, 3)$. We can directly evaluate the equation or locate the factor value by using the 8% interest table* and finding the factor of 1.2597 in the F/P column for $N = 3$.

Finding present worth of a future sum is simply the reverse of compounding and is known as *discounting process*. In Equation (17.2.1), we can see that if we were to find a present sum P, given a future sum F, we simply solve for P.

$$P = F \left[\frac{1}{(1 + i)^N} \right] = F(P/F, i, N) \quad (17.2.2)$$

The factor $1/(1 + i)^N$ is known as the *single payment present worth factor* and is designated $(P/F, i, N)$. Tables* have been constructed for the P/F factors for various values of i and N. The interest rate i and the P/F factor are also referred to as *discount rate* and *discounting factor*, respectively. Because using the interest tables is often the easiest way to solve an equation, this factor notation is included for each of the formulas derived in the following sections.

* All standard engineering economy textbooks (such as *Contemporary Engineering Economics* by C. S. Park, Addison Wesley, 1997) provide extensive sets of interest tables. Or you can obtain such interest tables on a World Wide Web site at <http://www.eng.auburn.edu/~park/cee.html>, which is a textbook web site for *Contemporary Engineering Economics*.

A Stream of Cash Flow Series

A common cash flow transaction involves a series of disbursements or receipts. Familiar situations such as car loans and insurance payments are examples of series payments. Payments of car loans and insurance bills typically involve identical sums paid at regular intervals. However, when there is no clear pattern of payment amounts over a series, one calls the transaction an *uneven cash-flow series*.

The present worth of any stream of payments can be found by calculating the present value of each individual payment and summing the results. Once the present worth is found, one can make other equivalence calculations, such as calculating the future worth by using the interest factors developed in the previous section.

Example 17.2.2 — Present Value of an Uneven Series by Decomposition into Single Payments

Wilson Technology, a growing machine shop, wishes to set aside money now to invest over the next 4 years in automating their customer service department. They now earn 10% on a lump sum deposited, and they wish to withdraw the money in the following increments:

- Year 1: \$25,000 to purchase a computer and data base software designed for customer service use
- Year 2: \$3000 to purchase additional hardware to accommodate anticipated growth in use of the system
- Year 3: No expenses
- Year 4: \$5000 to purchase software upgrades

How much money must be deposited now to cover the anticipated payments over the next 4 years?

Discussion. This problem is equivalent to asking what value of P would make you indifferent in your choice between P dollars today and the future expense stream of (\$25,000, \$3000, \$0, \$5000). One way to deal with an uneven series of cash flows is to calculate the equivalent present value of each single cash flow and sum the present values to find P . In other words, the cash flow is broken into three parts as shown in [Figure 17.2.2](#).

Solution.

$$\begin{aligned} P &= \$25,000(P/F, 10\%, 1) + \$3000(P/F, 10\%, 2) + \$5000(P/F, 10\%, 4) \\ &= \$28,622 \end{aligned}$$

Cash Flow Series with a Special Pattern

Whenever one can identify patterns in cash flow transactions, one may use them in developing concise expressions for computing either the present or future worth of the series. For this purpose, we will classify cash flow transactions into three categories: (1) equal cash flow series, (2) linear gradient series, and (3) geometric gradient series. To simplify the description of various interest formulas, we will use the following notation:

1. **Uniform Series:** Probably the most familiar category includes transactions arranged as a series of equal cash flows at regular intervals, known as an *equal-payment series* (or *uniform series*) ([Figure 17.2.3a](#)). This describes the cash flows, for example, of the common installment loan contract, which arranges for the repayment of a loan in equal periodic installments. The equal cash flow formulas deal with the equivalence relations of P , F , and A , the constant amount of the cash flows in the series.
2. **Linear Gradient Series:** While many transactions involve series of cash flows, the amounts are not always uniform: yet they may vary in some regular way. One common pattern of variation occurs when each cash flow in a series increases (or decreases) by a fixed amount ([Figure 17.2.3b](#)). A 5-year loan repayment plan might specify, for example, a series of annual payments that

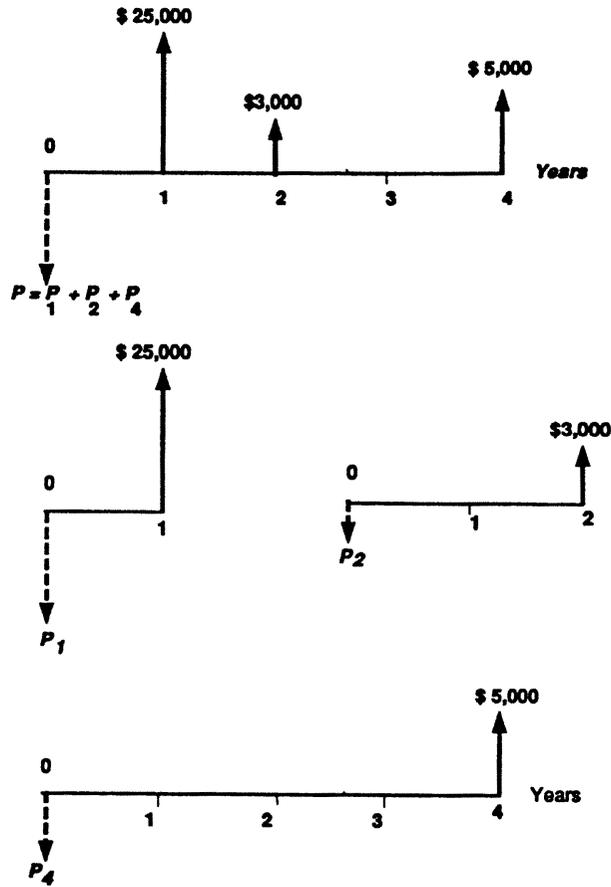


FIGURE 17.2.2 Decomposition of uneven cash flow series into three single-payment transactions. This decomposition allows us to use the single-payment present worth factor.

increased by \$50 each year. We call such a cash flow pattern a *linear gradient series* because its cash flow diagram produces an ascending (or descending) straight line. In addition to P, F, and A, the formulas used in such problems involve the constant amount, G, of the change in each cash flow.

3. **Geometric Gradient Series:** Another kind of gradient series is formed when the series in cash flow is determined, not by some fixed amount like \$50, but by some fixed *rate*, expressed as a percentage. For example, in a 5-year financial plan for a project, the cost of a particular raw material might be budgeted to increase at a rate of 4% per year. The curving gradient in the diagram of such a series suggests its name: a *geometric gradient series* (Figure 17.2.3c). In the formulas dealing with such series, the rate of change is represented by a lowercase g.

Table 17.2.1 summarizes the interest formulas and the cash flow situations in which they should be used. For example, the factor notation (F/A, i, N) represents the situation where you want to calculate the equivalent lump-sum future worth (F) for a given uniform payment series (A) over N period at interest rate i. Note that these interest formulas are applicable only when the interest (compounding) period is the same as the payment period. Also in this table we present some useful interest factor relationships. The next two examples illustrate how one might use these interest factors to determine the equivalent cash flow.

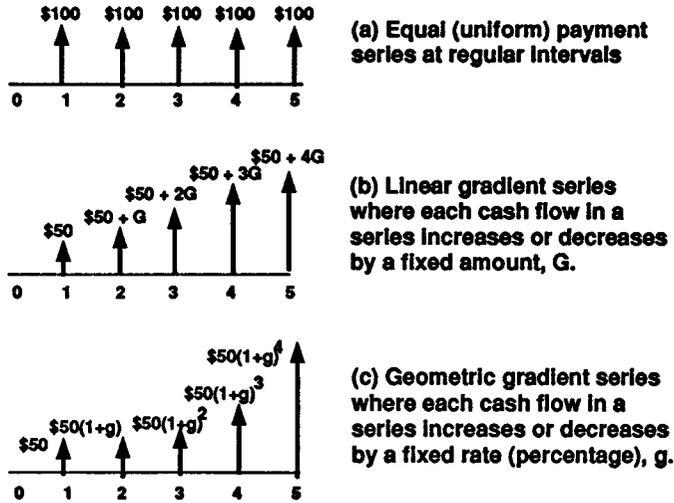


FIGURE 17.2.3 Five types of cash flows: (a) equal (uniform) payment series; (b) linear gradient series; and (c) geometric gradient series.

Example 17.2.3 — Uniform Series: Find F , Given i , A , N

Suppose you make an annual contribution of \$3000 to your savings account at the end of each year for 10 years. If your savings account earns 7% interest annually, how much can be withdrawn at the end of 10 years? (See Figure 17.2.4.)

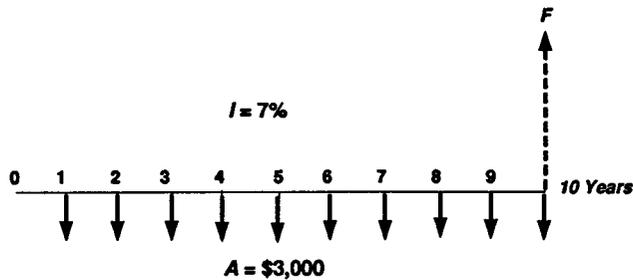


FIGURE 17.2.4 Cash flow diagram (Example 17.2.3).

Solution.

$$\begin{aligned}
 F &= \$3000(F/A, 7\%, 10) \\
 &= \$3000(13.8164) \\
 &= \$41,449.20
 \end{aligned}$$

Example 17.2.4 — Geometric Gradient: Find P , Given A_1 , g , i , N

Ansell Inc., a medical device manufacturer, uses compressed air in solenoids and pressure switches in the machines to control the various mechanical movements. Over the years the manufacturing floor has changed layouts numerous times. With each new layout more piping was added to the compressed air delivery system to accommodate the new locations of the manufacturing machines. None of the extra, unused old pipe was capped or removed; thus the current compressed air delivery system is inefficient and fraught with leaks. Because of the leaks in the current system, the compressor is expected to run

TABLE 17.2.1 Summary of Discrete Compounding Formulas with Discrete Payments

Flow Type	Factor Notation	Formula	Cash Flow Diagram	Factor Relationship
Single	Compound amount ($F/P, i, N$)	$F = P(1 + i)^N$		$(F/P, i, N) = i(F/A, i, N) + 1$ $(P/F, i, N) = 1 - (P/A, i, N)i$
	Present worth ($P/F, i, N$)	$P = F(1 + i)^{-N}$		
Equal Payment Series	Compound amount ($F/A, i, N$)	$F = A \left[\frac{(1+i)^N - 1}{i} \right]$		$(A/F, i, N) = (A/P, i, N) - i$ $(A/P, i, N) = \frac{i}{1 - (P/F, i, N)}$
	Sinking fund ($A/F, i, N$)	$A = F \left[\frac{i}{(1+i)^N - 1} \right]$		
	Present worth ($P/A, i, N$)	$P = A \left[\frac{(1+i)^N - 1}{i(1+i)^N} \right]$		
	Capital recovery ($A/P, i, N$)	$A = P \left[\frac{i(1+i)^N}{(1+i)^N - 1} \right]$		
Gradient Series	Uniform gradient Present worth ($P/G, i, N$)	$P = C \left[\frac{(1+i)^N - iN - 1}{i^2(1+i)^N} \right]$		$(F/G, i, N) = (P/G, i, N)(F/P, i, N)$ $(A/G, i, N) = (P/G, i, N)(A/P, i, N)$
	Geometric gradient Present worth ($P/A_1, g, i, N$)	$P = \begin{cases} A_1 \left[\frac{1 - (1+g)^N(1+i)^{-N}}{i-g} \right] \\ \frac{NA_1}{1+i} \quad (\text{if } i = g) \end{cases}$		$(F/A_1, g, i, N) = (P/A_1, g, i, N)(F/P, i, N)$

Adapted from Park, C.S. 1997. *Contemporary Engineering Economics*. Addison-Wesley, Reading, MA. Tables are constructed for various interest factors and you can obtain such interest tables on a World Wide Web site at <http://www.eng.auburn.edu/~park/cee.html>, which is a textbook web site for *Contemporary Engineering Economics*.

70% of the time that the plant is in operation during the upcoming year, which will require 260 kW/hr of electricity at a rate of \$0.05/kW-hr. (The plant runs 250 days a year for 24 hr a day.) If Ansell continues to operate the current air delivery system, the compressor run time will increase by 7% per year for the next 5 years due to ever-deteriorating leaks. (After 5 years, the current system cannot meet the plant's compressed air requirement, so it has to be replaced.) If Ansell decides to replace all of the old piping now, it will cost \$28,570. The compressor will still run the same number of days; however, it will run 23% less (or 70% $(1 - 0.23) = 53.9\%$ usage during the day) because of the reduced air pressure loss. If Ansell's interest rate is 12%, is it worth fixing now?

Solution.

- Step 1. Calculate the cost of power consumption of the current piping system during the first year. The power consumption is equal to:

$$\begin{aligned} \text{power cost} &= \% \text{ of day operating} \times \text{days operating per year} \times \text{hours per day} \\ &\quad \times \text{kW/hr} \times \$/\text{kW-hr} \\ &= (70\%) \times (250 \text{ days/year}) \times (24 \text{ hr/day}) \times (260 \text{ kW/hr}) \times (\$0.05/\text{kW-hr}) \\ &= \$54,440 \end{aligned}$$

- Step 2. Each year the annual power cost will increase at the rate of 7% over the previous year's power cost. Then the anticipated power cost over the 5-year period is summarized in [Figure 17.2.5](#). The equivalent present lump-sum cost at 12% for this geometric gradient series is

$$\begin{aligned} P_{old} &= \$54,440(P/A, 7\%, 12\%, 5) \\ &= \$54,440 \left[\frac{1 - (1 + 0.07)^5 (1 + 0.12)^{-5}}{0.12 - 0.07} \right] \\ &= \$222,283 \end{aligned}$$

- Step 3. If Ansell replaces the current compressed air system with the new one, the annual power cost will be 23% less during the first year and will remain at that level over the next 5 years. The equivalent present lump-sum cost at 12% is

$$\begin{aligned} P_{new} &= \$54,440(1 - 0.23)(P/A, 12\%, 5) \\ &= \$41,918.80(3.6048) \\ &= \$151,108 \end{aligned}$$

- Step 4. The net cost for not replacing the old system now is \$71,175 ($= \$222,283 - \$151,108$). Since the new system costs only \$28,570, the replacement should be made now.

Nominal and Effective Interest Rates

In all our examples in the previous section, we implicitly assumed that payments are received once a year, or annually. However, some of the most familiar financial transactions in both personal financial matters and engineering economic analysis involve nonannual payments; for example, monthly mortgage payments and daily earnings on savings accounts. Thus, if we are to compare different cash flows with different compounding periods, we need to address them on a common basis. The need to do this has led to the development of the concepts of *nominal interest rate* and *effective interest rate*.

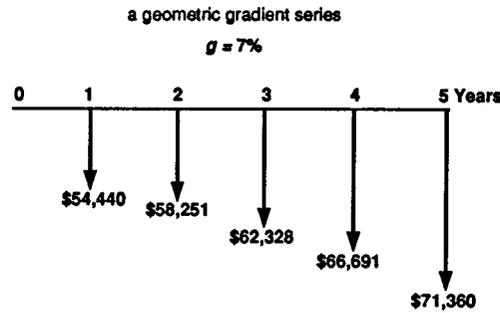


FIGURE 17.2.5 Expected power expenditure over the next 5 years due to deteriorating leaks if no repair is performed (Example 17.2.4).

Nominal Interest Rate

Even if a financial institution uses a unit of time other than a year — a month or quarter, for instance — in calculating interest payments, it usually quotes the interest rate on an annual basis. Many banks, for example, state the interest arrangement for credit cards in this way: “18% compounded monthly.” We say 18% is the *nominal interest rate* or *annual percentage rate* (APR), and the compounding frequency is monthly (12). To obtain the interest rate per compounding period, we divide 18% by 12 to obtain 1.5% per month. Therefore, the credit card statement above means that the bank will charge 1.5% interest on unpaid balance for each month.

Although the annual percentage rate, or APR, is commonly used by financial institutions and is familiar to customers, when compounding takes place more frequently than annually, the APR does not explain precisely the amount of interest that will accumulate in a year. To explain the true effect of more frequent compounding on annual interest amounts, we need to introduce the term effective interest rate.

Effective Annual Interest Rate

The *effective interest rate* is the only one that truly represents the interest earned in a year or some other time period. For instance, in our credit card example, the bank will charge 1.5% interest on unpaid balance at the end of each month. Therefore, the 1.5% rate represents an effective interest rate — on a monthly basis, it is the rate that predicts the actual interest payment on your outstanding credit card balance.

Suppose you purchase an appliance on credit at 18% compounded monthly. Unless you pay off the entire amount within a grace period (let’s say, a month), any unpaid balance (P) left for a year period would grow to

$$\begin{aligned}
 F &= P(1 + i)^N \\
 &= P(1 + 0.015)^{12} \\
 &= 1.1956P
 \end{aligned}$$

This implies that for each dollar borrowed for 1 year, you owe \$1.1956 at the end of the year, including the principal and interest. For each dollar borrowed, you pay an equivalent annual interest of 19.56 cents. In terms of an effective annual interest rate (i_a), we can rewrite the interest payment as a percentage of the principal amount

$$i_a = (1 + 0.015)^{12} - 1 = 0.1956, \text{ or } 19.56\%$$

Thus, the effective annual interest rate is 19.56%.

Clearly, compounding more frequently increases the amount of interest paid for the year at the same nominal interest rate. We can generalize the result to compute the effective interest rate for *any time duration*. As you will see later, we normally compute the effective interest rate based on payment (transaction) period. For example, cash flow transactions occur quarterly but interest rate is compounded monthly. This quarterly conversion allows us to use the interest formulas in [Table 17.2.1](#). To consider this, we may define the effective interest rate for a given payment period as

$$\begin{aligned} i &= (1 + r/M)^C - 1 \\ &= (1 + r/CK)^C - 1 \end{aligned} \tag{17.2.3}$$

where

- M = the number of interest periods per year
- C = the number of interest periods per payment period
- K = the number of payment periods per year

When $M = 1$, we have the special case of annual compounding. Substituting $M = 1$ into Equation (17.2.3), we find it reduces to $i_a = r$. That is, when compounding takes place once annually,* effective interest is equal to nominal interest. Thus, in all our earlier examples, where we considered only annual interest, we were by definition using effective interest rates.

Example 17.2.5 — Calculating Auto Loan Payments

Suppose you want to buy a car priced \$22,678.95. The car dealer is willing to offer a financing package with 8.5% annual percentage rate over 48 months. You can afford to make a down payment of \$2678.95, so the net amount to be financed is \$20,000. What would be the monthly payment?

Solution. The ad does not specify a compounding period, but in automobile financing the interest and the payment periods are almost always both monthly. Thus, the 8.5% APR means 8.5% compounded monthly. In this situation, we can easily compute the monthly payment using the *capital recovery factor* in [Table 17.2.1](#):

$$\begin{aligned} i &= 8.5\%/12 = 0.7083\% \text{ per month} \\ N &= (12)(4) = 48 \text{ months} \\ A &= \$20,000(A/P, 0.7083\%, 48) = \$492.97 \end{aligned}$$

* For an extreme case, we could consider a continuous compounding. As the number of compounding periods (M) becomes very large, the interest rate per compounding period (r/M) becomes very small. As M approaches infinity and r/M approaches zero, we approximate the situation of *continuous compounding*.

$$i = e^{r/K} - 1$$

To calculate the effective annual interest rate for continuous compounding, we set K equal to 1, resulting in:

$$i_a = e^r - 1$$

As an example, the effective annual interest rate for a nominal interest rate of 12% compounded continuously is $i_a = e^{0.12} - 1 = 12.7497\%$.

Example 17.2.6 — Compounding More Frequent Than Payments

Suppose you make equal quarterly deposits of \$1000 into a fund that pays interest at a rate of 12% compounded monthly. Find the balance at the end of year 2 (Figure 17.2.6).

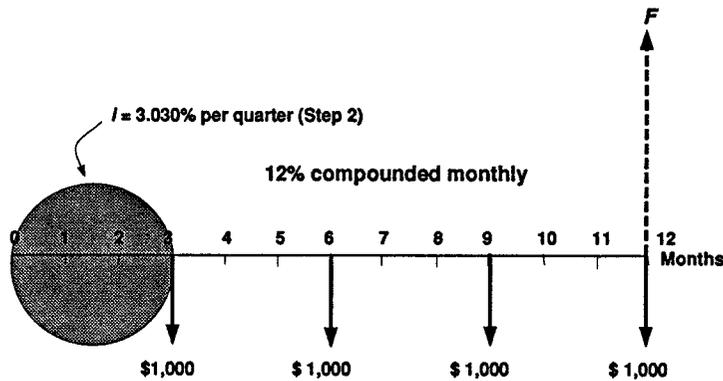


FIGURE 17.2.6 Quarterly deposits with monthly compounding (Example 17.2.6).

Solution. We follow the procedure for noncomparable compounding and payment periods described above.

1. Identify the parameter values for M, K, and C:

$$M = 12 \text{ compounding periods per year}$$

$$K = 4 \text{ payment periods per year}$$

$$C = 3 \text{ interest periods per payment period}$$

2. Use Equation (17.2.3) to compute effective interest per quarter.

$$i = (1 + 0.12/12)^3 - 1$$

$$= 3.030\% \text{ per quarter}$$

3. Find the total number of payment periods, N.

$$N = K(\text{number of years}) = 4(2) = 8 \text{ quarters}$$

4. Use i and N in the (F/A, i, N) factor from Table 17.2.1:

$$F = \$1000(F/A, 3.030\%, 8) = \$8901.81$$

Loss of Purchasing Power

It is important to differentiate between the time value of money as we used it in the previous section and the effects of inflation. The notion that a sum is worth more the earlier it is received can refer to its earning potential over time, to decreases in its value due to inflation over time, or to both. Historically, the general economy has usually fluctuated in such a way that it experiences *inflation*, a loss in the purchasing power of money over time. Inflation means that the cost of an item tends to increase over time or, to put it another way, the same dollar amount buys less of an item over time. *Deflation* is the

opposite of inflation (negative inflation), in that prices decrease over time and hence a specified dollar amount gains in purchasing power. In economic equivalence calculations, we need to consider the change of purchasing power along with the earning power.

The Average Inflation Rate

To account for the effect of varying yearly inflation rates over a period of several years, we can compute a single rate that represents an *average inflation rate*. Since each individual year's inflation rate is based on the previous year's rate, they have a compounding effect. As an example, suppose we want to calculate the average inflation rate for a 2-year period for a typical item. The first year's inflation rate is 4% and the second year's is 8%, using a base price index of 100.

- Step 1. We find the price at the end of the second year by the process of compounding:

$$\underbrace{100(1 + 0.04)}(1 + 0.08) = 112.32$$

- Step 2. To find the average inflation rate f over a 2-year period, we establish the following equivalence equation:

$$100(1 + f)^2 = 112.32 \leftarrow 100(F/P, f, 2) = 112.32$$

Solving for f yields

$$f = 5.98\%$$

We can say that the price increases in the last 2 years are equivalent to an average annual percentage rate of 5.98% per year. In other words, our purchasing power decreases at the annual rate of 5.98% over the previous year's dollars. If the average inflation rate is calculated based on the *consumer price index* (CPI), it is known as a *general inflation rate* (\bar{f}).

Actual vs. Constant Dollars

To introduce the effect of inflation into our economic analysis, we need to define several inflation-related terms.*

- Actual (current) dollars (A_n): Estimates of future cash flows for year n which take into account any anticipated changes in amount due to inflationary or deflationary effects. Usually these amounts are determined by applying an inflation rate to base year dollar estimates.
- Constant (real) dollars (A'_n): Dollars of constant purchasing power independent of the passage of time. In situations where inflationary effects have been assumed when cash flows were estimated, those estimates can be converted to constant dollars (base year dollars) by adjustment using some readily accepted *general inflation rate*. We will assume that the base year is always time 0 unless we specify otherwise.

* Based on the ANSI Z94 Standards Committee on Industrial Engineering Terminology. 1988. *The Engineering Economist*. Vol. 33(2): 145–171.

Equivalence Calculation under Inflation

In previous sections, our equivalence analyses have taken into consideration changes in the *earning power* of money — that is, interest effects. To factor in changes in *purchasing power* as well — that is, inflation — we may use either (1) constant dollar analysis or (2) actual dollar analysis. Either method will produce the same solution; however, each uses a different interest rate and procedure.

There are two types of interest rate for equivalence calculation: (1) the market interest rate and (2) the inflation-free interest rate. The interest rate that is applicable depends on the assumptions used in estimating the cash flow.

- Market interest rate (i): This interest rate takes into account the combined effects of the earning value of capital (earning power) and any anticipated inflation or deflation (purchasing power). Virtually all interest rates stated by financial institutions for loans and savings accounts are market interest rates. Most firms use a market interest rate (also known as *inflation-adjusted rate of return* [*discount rate*]) in evaluating their investment projects.
- Inflation-free interest rate (i'): An estimate of the true earning power of money when inflation effects have been removed. This rate is commonly known as *real interest rate* and can be computed if the market interest rate and inflation rate are known.

In calculating any cash flow equivalence, we need to identify the nature of project cash flows. There are three common cases:

- Case 1. All cash flow elements are estimated in constant dollars. Then, to find the equivalent present value of a cash flow series in constant dollars, use the inflation-free interest rate.
- Case 2. All cash flow elements are estimated in actual dollars. Then, use the market interest rate to find the equivalent worth of the cash flow series in actual dollars.
- Case 3. Some of the cash flow elements are estimated in constant dollars and others are estimated in actual dollars. In this situation, we simply convert all cash flow elements into one type — either constant or actual dollars. Then we proceed with either constant-dollar analysis for case 1 or actual-dollar analysis for case 2.

Removing the effect of inflation by deflating the actual dollars with \bar{f} and finding the equivalent worth of these constant dollars by using the inflation-free interest rate can be greatly streamlined by the efficiency of the *adjusted-discount method*, which performs deflation and discounting in one step. Mathematically we can combine this two-step procedure into one by

$$i = i' + \bar{f} + i'\bar{f} \quad (17.2.4)$$

This implies that the market interest rate is a function of two terms, i' , \bar{f} . Note that if there is no inflationary effect, the two interest rates are the same ($\bar{f} = 0 \rightarrow i = i'$). As either i' or \bar{f} increases, i also increases. For example, we can easily observe that when prices are increasing due to inflation, bond rates climb, because lenders (that is anyone who invests in a money-market fund, bond, or certificate of deposit) demand higher rates to protect themselves against the eroding value of their dollars. If inflation were at 3%, you might be satisfied with an interest rate of 7% on a bond because your return would more than beat inflation. If inflation were running at 10%, however, you would not buy a 7% bond; you might insist instead on a return of at least 14%. On the other hand, when prices are coming down, or at least are stable, lenders do not fear the loss of purchasing power with the loans they make, so they are satisfied to lend at lower interest rates.

17.3 Measures of Project Worth

This section shows how to compare alternatives on equal basis and select the wisest alternative from an economic standpoint. The three common measures based on cash flow equivalence are (1) equivalent present worth, (2) equivalent annual worth, and (3) rate of return. The present worth represents a measure of future cash flow relative to the time point “now” with provisions that account for earning opportunities. Annual worth is a measure of the cash flow in terms of the equivalent equal payments on an annual basis. The third measure is based on *yield* or percentage.

Describing Project Cash Flows

When a company purchases a fixed asset such as equipment, it makes an investment. The company commits funds today in the expectation of earning a return on those funds in the future. Such an investment is similar to that made by a bank when it lends money. For the bank loan, the future cash flow consists of interest plus repayment of the principal. For the fixed asset, the future return is in the form of cash flows from the profitable use of the asset. In evaluating a capital investment, we are concerned only with those cash flows that result directly from the investment. These cash flows, called *differential* or *incremental cash flows*, represent the change in the firm’s total cash flow that occurs as a direct result of the investment.

We must also recognize that one of the most important parts of the capital budgeting process is the estimation of the relevant cash flows. For all examples in this section, however, net cash flows can be viewed as before-tax values or after-tax values for which tax effects have been recalculated. Since some organizations (e.g., governments and nonprofit organizations) are not taxable, the before-tax situation can be a valid base for that type of economic evaluation. This view will allow us to focus on our main area of concern, the economic evaluation of an investment project. The procedures for determining after-tax net cash flows in taxable situations are developed in Section 17.4.

Example 17.3.1 — Identifying Project Cash Flows

Merco Inc., a machinery builder in Louisville, KY, is considering making an investment of \$1,250,000 in a complete structural-beam-fabrication system. The increased productivity resulting from the installation of the drilling system is central to the justification. Merco estimates the following figures as a basis for calculating productivity:

- Increased fabricated steel production: 2000 tons/year
- Average sales price/ton fabricated steel: \$2566.50/ton
- Labor rate: \$10.50/hr
- Tons of steel produced in a year: 15,000 tons
- Cost of steel per ton (2205 lb): \$1950/ton
- Number of workers on layout, holmaking, sawing, and material handling: 17
- Additional maintenance cost: \$128,500 per year

With the cost of steel at \$1950/ton and the direct labor cost of fabricating 1 lb at 10 cents, the cost of producing a ton of fabricated steel is about \$2170.50. With a selling price of \$2566.50/ton, the resulting contribution to overhead and profit becomes \$396/ton. Assuming that Merco will be able to sustain an increased production of 2000 tons per year by purchasing the system, the projected additional contribution has been estimated to be $2000 \text{ tons} \times \$396 = \$792,000$.

Since the drilling system has the capacity to fabricate the full range of structural steel, two workers can run the system, one on the saw and the other on the drill. A third operator is required as a crane operator for loading and unloading materials. Merco estimates that to do the equivalent work of these

three workers with conventional manufacture requires, on the average, an additional 14 people for centerpunching, holmaking with radial or magnetic drill, and material handling. This translates into a labor savings in the amount of \$294,000 per year ($\$10.50 \times 40 \text{ hr/week} \times 50 \text{ weeks/year} \times 14$). The system can last for 15 years with an estimated after-tax salvage value of \$80,000. The expected annual corporate income taxes would amount to \$226,000. Determine the net cash flow from undertaking the investment. Determine the net cash flows from the project over the service life.

Solution. The net investment cost as well as savings are as follows:

- Cash inflows:
 - Increased annual revenue: \$792,000
 - Projected annual net savings in labor: \$294,000
 - Projected after-tax salvage value at the end of year 15: \$80,000
- Cash outflows:
 - Project investment cost: \$1,250,000
 - Projected increase in annual maintenance cost: \$128,500
 - Projected increase in corporate income taxes: \$226,000

Now we are ready to summarize a cash flow table as follows:

Year	Cash Inflows	Cash Outflows	Net Cash Flows
0	0	\$1,250,000	-\$1,250,000
1	1,086,000	354,500	731,500
2	1,086,000	354,500	731,500
⋮	⋮	⋮	⋮
15	1,086,000 + 80,000	354,500	811,500

The project’s cash flow diagram is shown in [Figure 17.3.1](#).

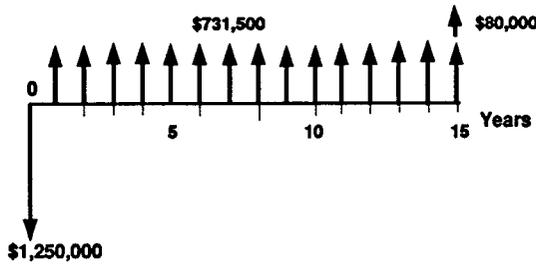


FIGURE 17.3.1 Cash flow diagram (Example 17.3.1).

Assuming these cost savings and cash flow estimates are correct, should management give the go-ahead for installation of the system? If management has decided not to install the fabrication system, what do they do with the \$1,250,000 (assuming they have it in the first place)? The company could buy \$1,250,000 of Treasury bonds. Or it could invest the amount in other cost-saving projects. How would the company compare cash flows that differ both in timing and amount for the alternatives it is considering? This is an extremely important question because virtually every engineering investment decision involves a comparison of alternatives. These are the types of questions this section is designed to help you answer.

Present Worth Analysis

Until the 1950s, the paycheck method* was widely used as a means of making investment decisions. As flows in this method were recognized, however, business people began to search for methods to improve project evaluations. This led to the development of discounted cash flow techniques (DCF), which take into account the time value of money. One of the DCFs is the net present worth method (NPW). A capital investment problem is essentially one of determining whether the anticipated cash inflows from a proposed project are sufficiently attractive to invest funds in the project. In developing the NPW criterion, we will use the concept of cash flow equivalence discussed in Section 17.2. Usually, the most convenient point at which to calculate the equivalent values is often time 0. Under the NPW criterion, the present worth of all cash inflows is compared against the present worth of all cash outflows that are associated with an investment project. The difference between the present worth of these cash flows, called the *net present worth* (NPW), determines whether or not the project is an acceptable investment. When two or more projects are under consideration, NPW analysis further allows us to select the best project by comparing their NPW figures.

We will first summarize the basic procedure for applying the present worth criterion to a typical investment project.

- Determine the interest rate that the firm wishes to earn on its investments. This represents an interest rate at which the firm can always invest the money in its *investment pool*. We often refer to this interest rate as either a *required rate of return* or a *minimum attractive rate of return* (MARR). Usually this selection will be a policy decision by top management. It is possible for the MARR to change over the life of a project, but for now we will use a single rate of interest in calculating NPW.
- Estimate the service life of the project.**
- Determine the net cash flows (net cash flow = cash inflow – cash outflow).
- Find the present worth of each net cash flow at the MARR. Add up these present worth figures; their sum is defined as the project's NPW.
- Here, a positive NPW means the equivalent worth of inflows are greater than the equivalent worth of outflows, so project makes a profit. Therefore, if the $PW(i)$ is positive for a single project, the project should be accepted; if negative, it should be rejected. The decision rule is

If $PW(i) > 0$, accept the investment

If $PW(i) = 0$, remain indifferent

If $PW(i) < 0$, reject the investment

Note that the decision rule is for a single project evaluation where you can estimate the revenues as well as costs associated with the project. As you will find later, when you are comparing alternatives with

* One of the primary concerns of most business people is whether and when the money invested in a project can be recovered. The *payback method* screens projects on the basis of how long it takes for net receipts to equal investment outlays. A common standard used in determining whether or not to pursue a project is that no project may be considered unless its payback period is shorter than some specified period of time. If the payback period is within the acceptable range, a formal project evaluation (such as the present worth analysis) may begin. It is important to remember that payback screening is not an *end* itself, but rather a method of screening out certain obvious unacceptable investment alternatives before progressing to an analysis of potentially acceptable ones. But the much-used payback method of equipment screening has a number of serious drawbacks. The principal objection to the payback method is its failure to measure profitability; that is, there is no “profit” made during the payback period. Simply measuring how long it will take to recover the initial investment outlay contributes little to gauging the earning power of a project.

the same revenues, you can compare them based on the cost only. In this situation (because you are minimizing costs, rather than maximizing profits), you should accept the project that results in smallest, or least negative, NPW.

Example 17.3.2 — Net Present Worth

Consider the investment cash flows associated with the metal fabrication project in Example 17.3.1. If the firm’s MARR is 15%, compute the NPW of this project. Is this project acceptable?

Solution. Since the fabrication project requires an initial investment of \$1,250,000 at $n = 0$ followed by the 15 equal annual savings of \$731,500, and \$80,000 salvage value at the end of 15 years, we can easily determine the NPW as follows:

$$\begin{aligned}
 PW(15\%)_{outflow} &= \$1,250,000 \\
 PW(15\%)_{inflow} &= \$731,500(P/A, 15\%, 15) + \$80,000(P/F, 15\%, 15) \\
 &= \$4,284,259
 \end{aligned}$$

Then, the NPW of the project is

$$\begin{aligned}
 PW(15\%) &= PW(15\%)_{inflow} - PW(15\%)_{outflow} \\
 &= \$4,284,259 - \$1,250,000 \\
 &= \$3,034,259
 \end{aligned}$$

Since $PW(15\%) > 0$, the project would be acceptable.

Annual Equivalent Method

The annual equivalent worth (AE) criterion is a basis for measuring investment worth by determining equal payments on an annual basis. Knowing that we can convert any lump-sum cash amount into a series of equal annual payments, we may first find the NPW for the original series and then multiply the NPW by the capital recovery factor:

** Another special case of the PW criterion is useful when the life of a proposed project is *perpetual* or the planning horizon is extremely long. The process of computing the PW cost for this infinite series is referred to as the *capitalization* of project cost. The cost is known as the *capitalized cost*. It represents the amount of money that must be invested today to yield a certain return A at the end of each and every period forever, assuming an interest rate of i . Observe the limit of the uniform series present worth factor as N approaches infinity:

$$\lim_{N \rightarrow \infty} (P/A, i, N) = \lim_{N \rightarrow \infty} \left[\frac{(1+i)^N - 1}{i(1+i)^N} \right] = \frac{1}{i}$$

Thus, it follows that

$$PW(i) = A(P/A, i, N \rightarrow \infty) = \frac{A}{i} \tag{17.5}$$

Another way of looking at this, $PW(i)$ dollars today, is to ask what constant income stream could be generated by this in perpetuity. Clearly, the answer is $A = iPW(i)$. If withdrawals were greater than A , they could be eating into the principal, which would eventually reduce to 0.

$$AE(i) = PW(i)(A/P, i, N) \quad (17.3.1)$$

The accept-reject decision rule for a single *revenue* project is

If $AE(i) > 0$, accept the investment

If $AE(i) = 0$, remain indifferent

If $AE(i) < 0$, reject the investment

Notice that the factor $(A/P, i, N)$ in [Table 17.2.1](#) is positive for $-1 < i < \infty$. This indicates that the $AE(i)$ value will be positive if and only if $PW(i)$ is positive. In other words, accepting a project that has a positive $AE(i)$ value is equivalent to accepting a project that has a positive $PW(i)$ value. Therefore, the AE criterion should provide a basis for evaluating a project that is consistent with the NPW criterion.

As with the present worth analysis, when you are comparing mutually exclusive *service* projects whose revenues are the same, you may compare them based on *cost* only. In this situation, you will select the alternative with the minimum annual equivalent cost (or least negative annual equivalent worth).

Unit Profit/Cost Calculation

There are many situations in which we want to know the unit profit (or cost) of operating an asset. A general procedure to obtain such a unit profit or cost figure involves the following two steps:

- Determine the number of units to be produced (or serviced) each year over the life of the asset.
- Identify the cash flow series associated with the production or service over the life of the asset.
- Calculate the net present worth of the project cash flow series at a given interest rate and then determine the equivalent annual worth.
- Divide the equivalent annual worth by the number of units to be produced or serviced during each year. When you have the number of units varying each year, you may need to convert them into equivalent annual units.

To illustrate the procedure, we will consider [Example 17.3.3](#), where the annual equivalent concept can be useful in estimating the savings per machine hour for a proposed machine acquisition.

Example 17.3.3 — Unit Profit per Machine Hour

Tiger Machine Tool Company is considering the proposed acquisition of a new metal-cutting machine. The required initial investment of \$75,000 and the projected cash benefits and annual operating hours over the 3-year project life are as follows.

End of Year	Net Cash Flow	Operating Hours
0	-\$75,000	
1	24,400	2,000
2	27,340	2,000
3	55,760	2,000

Compute the equivalent savings per machine hour at $i = 15\%$.

Solution. Bringing each flow to its equivalent at time zero, we find

$$\begin{aligned} PW(15\%) &= -\$75,000 + \$24,400(P/F, 15\%, 1) + \$27,340(P/F, 15\%, 2) + \$55,760(P/F, 15\%, 3) \\ &= \$3553 \end{aligned}$$

Since the project results in a surplus of \$3553, the project would be acceptable. We first compute the annual equivalent savings from the use of the machine. Since we already know the NPW of the project, we obtain the AE by

$$AE(15\%) = \$3553(A/P, 15\%, 3) = \$1556$$

With an annual usage of 2000 hr, the equivalent savings per machine hour would be

$$\text{Savings per machine hour} = \$1556/\$2000 \text{ hr} = \$0.78/\text{hr}$$

Comments. Note that we cannot simply divide the NPW amount (\$3553) by the total number of machine hours over the 3-year period (6000 hr), or \$0.59/hr. This \$0.59 figure represents the *instant savings* in present worth for each hourly use of the equipment, but does not consider the time over which the savings occur. Once we have the annual equivalent worth, we can divide by the desired time unit if the compounding period is 1 year. If the compounding period is shorter, then the equivalent worth should be calculated for the compounding period.

Rate of Return Analysis

Along with the NPW and AE, the third primary measure of investment worth is based on yield, known as *rate of return*. The NPW measure is easy to calculate and apply. Nevertheless, many engineers and financial managers prefer rate of return analysis to the NPW method because they find it intuitively more appealing to analyze investments in terms of percentage rates of return than in dollars of NPW.

Internal Rate of Return

Many different terms refer to rate of return, including yield (that is, the yield to maturity, commonly used in bond valuation), internal rate of return, and marginal efficiency of capital. In this section, we will define *internal rate of return* as the break-even interest rate, i^* , which equates the present worth of a project's cash outflows to the present worth of its cash inflows, or

$$\begin{aligned} PW(i^*) &= PW_{\text{cash inflows}} - PW_{\text{cash outflows}} \\ &= 0 \end{aligned}$$

Note that the NPW expression is equivalent to

$$PW(i^*) = \frac{A_0}{(1+i^*)^0} + \frac{A_1}{(1+i^*)^1} + \dots + \frac{A_N}{(1+i^*)^N} = 0 \quad (17.3.2)$$

Here we know the value of A_n for all n , but not the value of i^* . Since it is the only unknown, we can solve for i^* . There will inevitably be N values of i^* that satisfy this equation. In most project cash flows you would be able to find a unique positive i^* that satisfies Equation (17.3.2). However, you may encounter some cash flow that cannot be solved for a single rate of return greater than -100% . By the nature of the NPW function in Equation (17.3.2), it is certainly possible to have more than one rate of return for a certain type of cash flow.* (For some cash flows, we may not find any rate of return at all.)

Finding the IRR

We don't need laborious manual calculations to find i^* . Many financial calculators have built-in functions for calculating i^* . It is also worth noting here that many spreadsheet packages have i^* functions, which solve Equation (17.3.2) very rapidly. This is normally done by entering the cash flows through a computer

keyboard or by reading a cash flow data file. As an alternative, you could try the trial-and-error method to locate the break-even interest that makes the net present worth equal to zero.

Accept/Reject Decision Rules

Why are we interested in finding the particular interest rate that equates a project's cost with the present worth of its receipts? Again, we may easily answer this by examining Figure 17.3.2. In this figure, we notice two important characteristics of the NPW profile. First, as we compute the project's $PW(i)$ at a varying interest rate (i), we see that the NPW becomes positive for $i < i^*$, indicating that the project would be acceptable under the PW analysis for those values of i . Second, the NPW becomes negative for $i > i^*$, indicating that the project is unacceptable for those values of i . Therefore, the i^* serves as a *break-even* interest rate. By knowing this break-even rate, we will be able to make an accept/reject decision that is consistent with the NPW analysis.

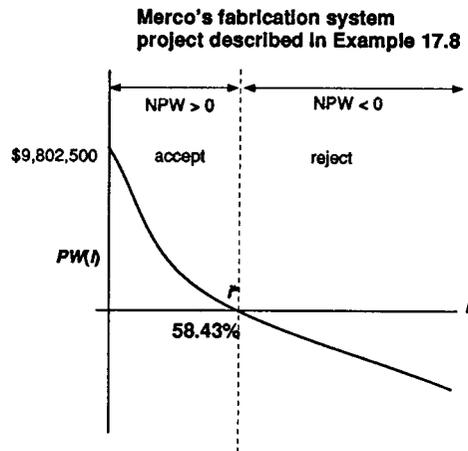


FIGURE 17.3.2 A net present worth profile for the cash flow series given in Figure 17.3.1 at varying interest rates. The project breaks even at 58.43% so that the NPW will be positive as long as the discount rate is less than 58.43%.

At the MARR the company will more than break even. Thus, the IRR becomes a useful gauge against which to judge project acceptability, and the decision rule for a simple project is

* When applied to projects that require investments at the outset followed by a series of cash inflows (or a simple project), the i^* provides an unambiguous criterion for measuring profitability. However, when multiple rates of return occur, none of them is an accurate portrayal of project acceptability or profitability. Clearly, then, we should place a high priority on discovering this situation early in our analysis of a project's cash flows. The quickest way to predict multiple i^* s is to generate a NPW profile and check to see if it crosses the horizontal axis more than once.

In addition to the NPW profile, there are good — although somewhat more complex — analytical methods for predicting multiple i^* s. Perhaps more importantly, there is a good method, which uses a *cost of capital*, of refining our analysis when we do discover multiple i^* s. Use of a cost of capital allows us to calculate a single accurate rate of return (also known as *return on invested capital*); it is covered in *Contemporary Engineering Economics*, C.S. Park, Addison-Wesley, 1997. If you choose to avoid these more complex applications of rate-of-return techniques, you must at a minimum be able to predict multiple i^* s via the NPW profile and, when they occur, select an alternative method such as NPW or AE analysis for determining project acceptability.

- If $IRR > MARR$, accept the project
- If $IRR = MARR$, remain indifferent
- If $IRR < MARR$, reject the project

Note that this decision rule is designed to be applied for a single project evaluation. When we have to compare mutually exclusive investment projects, we need to apply the incremental analysis, as we shall see in a later section.

Example 17.3.4 — Rate of Return Analysis

Reconsider the fabrication investment project in Example 17.3.1. (a) What is the projected IRR on this fabrication investment? (b) If Merco's MARR is known to be 15%, is this investment justifiable?

Solution.

- (a) The net present worth expression as a function of interest rate (i) is

$$PW(i) = -\$1,250,000 + \$731,500(P/A, i, 15) + \$80,000(P/F, i, 15) \\ = 0$$

Using Excel's financial function (IRR), we find the IRR to be 58.43%. (See [Figure 17.3.2](#).) Merco will recover the initial investment fully and also earn 58.43% interest on its invested capital.

- (b) If Merco does not undertake the project, the \$1,250,000 would remain in the firm's investment pool and continue to earn only 15% interest. The IRR figure far exceeds the Merco's MARR, indicating that the fabrication system project is an economically attractive one. Merco's management believes that, over a broad base of structural products, there is no doubt that the installation of its fabricating system would result in a significant savings, even after considering some potential deviations from the estimates used in the analysis.

Mutually Exclusive Alternatives

Until now, we have considered situations in which only one project was under consideration, and we were determining whether to pursue it, based on whether its present worth or rate of return met our MARR requirements. We were making an accept or reject decision about a *single* project.

In the real world of engineering practice, however, it is more typical for us to have two or more choices of projects for accomplishing a business objective. *Mutually exclusive* means that any one of several alternatives will fulfill the same need and that selecting one alternative means that the others will be excluded.

Analysis Period

The *analysis period* is the time span over which the economic effects of an investment will be evaluated. The analysis period may also be called the *study period* or *planning horizon*. The length of the analysis period may be determined in several ways: it may be a predetermined amount of time set by company policy, or it may be either implied or explicit in the need the company is trying to fulfill. In either of these situations, we consider the analysis period to be a *required service period*. When no required service period is stated at the outset, the analyst must choose an appropriate analysis period over which to study the alternative investment projects. In such a case, one convenient choice of analysis period is the period of useful life of the investment project.

When useful life of the investment project does not match the analysis or required service period, we must make adjustments in our analysis. A further complication, when we are considering two or more mutually exclusive projects, is that the investments themselves may have differing useful lives. We must

compare projects with different useful lives over an *equal time span*, which may require further adjustments in our analysis.

Analysis Period Equals Project Lives

Let's begin our analysis with the simplest situation where the project lives equal the analysis period. In this case, we compute the NPW for each project and select the one with highest NPW for revenue projects or least negative NPW for service projects. Example 17.3.5 will illustrate this point.

Example 17.3.5 — Two Mutually Exclusive Alternatives

A pilot wants to start her own company to airlift goods to the Commonwealth of Independent States (formerly the U.S.S.R.) during their transition to a free-market economy. To economize the start-up business, she decides to purchase only one plane and fly it herself. She has two mutually exclusive options: an old aircraft (A1) or a new jet (A2) with which she expects to have higher purchase costs, but higher revenues as well because of its larger payload. In either case, she expects to fold up business in 3 years because of competition from larger companies. The cash flows for the two mutually exclusive alternatives are given in thousand dollars:

<i>n</i>	A1	A2
0	-3,000	-\$12,000
1	1,350	4,200
2	1,800	6,225
3	1,500	6,330

Assuming that there is no do-nothing alternative, which project would she select at MARR = 10%?

Solution. Since the required service period is 3 years, we should select the analysis period of 3 years. Since the analysis period coincides with the project lives, we simply compute the NPW value for each option. The equivalent NPW figures at $i = 10\%$ would be as follows:

- For A1:

$$\begin{aligned} PW(15\%)_{A1} &= -\$3000 + \$1350(P/F, 10\%, 1) + \$1800(P/F, 10\%, 2) + \$1500(P/F, 10\%, 3) \\ &= \$842 \end{aligned}$$

- For A2:

$$\begin{aligned} PW(15\%)_{A2} &= -\$12,000 + \$4200(P/F, 10\%, 1) + \$6225(P/F, 10\%, 2) + \$6330(P/F, 10\%, 3) \\ &= \$1719 \end{aligned}$$

Clearly, A2 is the most economical option.

Project Lives Differ from a Specified Analysis Period

Often project lives do not match the required analysis period and/or do not match each other. For example, two machines may perform exactly the same function, but one lasts longer than the other and both of them last longer than the analysis period for which they are being considered. We are then left with some unused portion of the equipment, which we include as salvage value in our analysis. Salvage value is the amount of money for which the equipment could be sold after its service or the dollar measure of its remaining usefulness.

When project lives are shorter than the required service period, we must consider how, at the end of the project lives, we will satisfy the rest of the required service period. Replacement projects — additional

projects to be implemented when the initial project has reached the limits of its useful life — are needed in such a case. Sufficient replacement projects must be analyzed to match or exceed the required service period.

To simplify our analysis, we sometimes assume that the replacement project will be exactly the same as the initial project, with the same corresponding costs and benefits. However, this assumption is not necessary. For example, depending on our forecasting skills, we may decide that a different kind of technology — in the form of equipment, materials, or processes — is a preferable replacement. Whether we select exactly the same alternative or a new technology as the replacement project, we are ultimately likely to have some unused portion of the equipment to consider as salvage value at the end of the required service period. On the other hand, if a required service period is relatively short, we may decide to lease the necessary equipment or subcontract the remaining work for the duration of the analysis period. In this case, we can probably exactly match our analysis period and not worry about salvage values.

Example 17.3.6 — Present Worth Comparison — Project Lives Shorter Than Analysis Period

The Smith Novelty Company, a mail-order firm, wants to install an automatic mailing system to handle product announcements and invoices. The firm has a choice between two different types of machines. The two machines are designed differently but have identical capacities and do exactly the same job. The \$12,500 semiautomatic model A will last 3 years with a salvage value of \$2000, while the fully automatic model B will cost \$15,000 and last 4 years with a salvage value of \$1500. The expected cash flows for the two machines including maintenance, salvage value, and tax effects are as follows:

<i>n</i>	Model A	Model B
0	-12,500	-\$15,000
1	-5,000	-4,000
2	-5,000	-4,000
3	-5,000 + 2,000	-4,000
4		-4,000 + 1,500
5		

As business grows to a certain level, neither of the models can handle the expanded volume at the end of year 5. If that happens, a fully computerized mail-order system will need to be installed to handle the increased business volume. With this scenario, which model should the firm select at MARR = 15%?

Solution. Since both models have a shorter life than the required service period (5 years), we need to make an explicit assumption of how the service requirement is to be met. Suppose that the company considers leasing comparable equipment that has an annual lease payment of \$6000 (after taxes) for the remaining required service period. In this case, the cash flow would look like [Figure 17.3.3](#).

<i>n</i>	Model A	Model B
0	-12,500	-\$15,000
1	-5,000	-4,000
2	-5,000	-4,000
3	-3,000	-4,000
4	-6,000 -5,000	-2,500
5	-6,000 -5,000	-6,000 -5,000

Here the bold figures represent the annual lease payments. (It costs \$6000 to lease the equipment and \$5000 to operate annually. Other maintenance will be paid by the leasing company.) Note that both alternatives now have the same required service period of 5 years. Therefore, we can use NPW analysis.

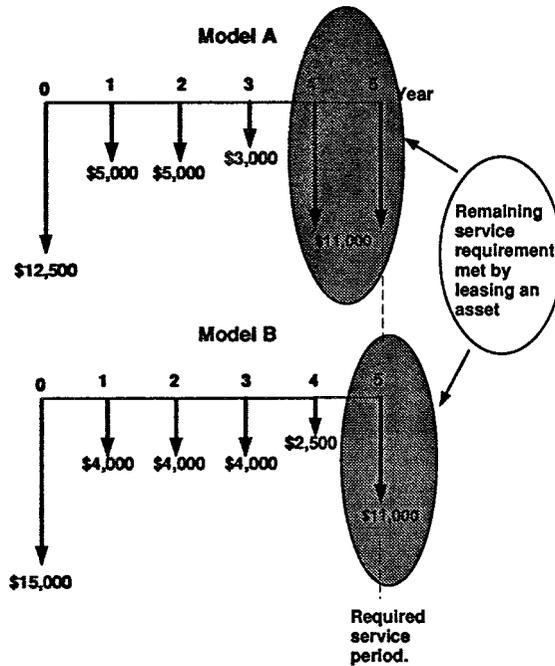


FIGURE 17.3.3 Comparison for unequal-lived projects when the required service period is longer than the individual project life (Example 17.3.6.)

$$\begin{aligned}
 PW(15\%)_A &= -\$12,500 - \$5000(P/A, 15\%, 2) + \$3000(P/F, 15\%, 3) \\
 &\quad - \$11,000(P/A, 15\%, 2)(P/F, 15\%, 3) \\
 &= -\$34,359
 \end{aligned}$$

$$\begin{aligned}
 PW(15\%)_B &= -\$15,000 - \$4000(P/A, 15\%, 3) - \$2500(P/F, 15\%, 4) - \$11,000(P/F, 15\%, 5) \\
 &= -\$32,747
 \end{aligned}$$

Since these are service projects, model B is the better choice.

Flaws in Project Ranking by IRR. Under NPW, the mutually exclusive project with the highest worth figure was preferred. Unfortunately, the analogy does not carry over to IRR analysis. The project with the highest IRR may *not* be the preferred alternative. To illustrate the flaws of comparing IRRs to choose from mutually exclusive projects, suppose you have two mutually exclusive alternatives, each with a 1-year service life; one requires an investment of \$1000 with a return of \$2000 and the other requires \$5000 with a return of \$7000. You already obtained the IRRs and NPWs at MARR = 10% as follows:

<i>n</i>	A1	A2
0	-\$1000	-\$5000
1	222000	7000
IRR	100%	40%
PW(10%)	\$818	\$1364

Would you prefer the first project simply because you expect a higher rate of return?

We can see that A2 is preferred over A1 by the NPW measure. On the other hand, the IRR measure gives a *numerically* higher rating for A1. This inconsistency in ranking is due to the fact that the NPW is an *absolute (dollar)* measure of investment worth, while the IRR is a *relative (percentage)* measure and cannot be applied in the same way. That is, the IRR measure ignores the *scale* of the investment. Therefore, the answer is no; instead, you would prefer the second project with the lower rate of return, but higher NPW. The NPW measure would lead to that choice, but comparison of IRRs would rank the smaller project higher. Another approach, called *incremental analysis*, is needed.

Rate of Return on Incremental Investment

In our previous ranking example, the more costly option requires an incremental investment of \$4000 at an incremental return of \$5000. If you decide to take the more costly option, certainly you would be interested in knowing that this additional investment can be justified at the MARR. The 10% of MARR value implies that you can always earn that rate from other investment sources — \$4400 at the end of 1 year for \$4000 investment. However, by investing the additional \$4000 in the second option, you would make an additional \$5000, which is equivalent to earning at the rate of 25%. Therefore, the incremental investment can be justified.

Now we can generalize the decision rule for comparing mutually exclusive projects. For a pair of mutually exclusive projects (A, B), rate of return analysis is done by computing the *internal rate of return on incremental investment* (IRR_{Δ}) between the projects. Since we want to consider increments of investment, we compute the cash flow for the difference between the projects by subtracting the cash flow for the lower investment-cost project (A) from that of the higher investment-cost project (B). Then, the decision rule is select B, if $IRR_{B-A} > MARR$. Otherwise select A.

Example 17.3.7 — IRR on Incremental Investment: Two Alternatives

Reconsider the two mutually exclusive projects in Example 17.3.5.

<i>n</i>	B1	B2	B2–B1
0	–\$3,000	–\$12,000	–\$9,000
1	1,350	4,200	2,850
2	1,800	6,225	4,425
3	<u>1,500</u>	<u>6,330</u>	4,830
IRR	25%	17.43%	

Since B1 is the lower cost investment project, we compute the incremental cash flow for B2–B1. Then we compute the IRR on this increment of investment by solving

$$-\$9000 + \$2850(P/F, i, 1) + \$4425(P/F, i, 2) + \$4830(P/F, i, 3) = 0$$

We obtain $i^*_{B2-B1} = 15\%$. Since $IRR_{B2-B1} > MARR$, we select B2, which is consistent with the NPW analysis.

Comments. Why did we choose to look at the increment B2-B1 instead of B1-B2? We want the increment to have investment during at least some part of the time span so that we can calculate an IRR. Subtracting the lower initial investment project from the higher guarantees that the first increment will be investment flow. Ignoring the investment ranking, we might end up with an increment which involves borrowing cash flow and has no internal rate of return. This is the case for B1-B2. (i^*_{B1-B2} is also 15%, not –15%.) If we erroneously compare this i^* with MARR, we might have accepted project B1 over B2.

17.4 Cash Flow Projections

With the purchase of any fixed asset such as equipment, we need to estimate the profits (more precisely, cash flows) that the asset will generate during its service period. An inaccurate estimate of asset needs can have serious consequences. If you invest too much in assets, you incur unnecessarily heavy expenses. Spending too little on fixed assets also is harmful, for then the firm's equipment may be too obsolete to produce products competitively and without an adequate capacity you may lose a portion of your market share to rival firms. Regaining lost customers involves heavy marketing expenses and may even require price reductions and/or product improvements, all of which are costly. We will begin this section by giving an overview on how a company determines its operating profit.

Operating Profit — Net Income

Firms invest in a project because they expect it to increase their wealth. If the project does this — if project revenues exceed project costs — we say it has generated a *profit*, or *income*. If the project reduces the owner's wealth, we say that the project has resulted in a *loss*. One of the most important roles of the accounting function within an organization is to measure the amount of profit or loss a project generates each year or in any other relevant time period. Any profit generated will be taxed. The accounting measure of a project's after-tax profit during a particular time period is known as *net income*.

Accountants measure the net income of a specified operating period by subtracting expenses from revenues for that period. These terms can be defined as follows:

1. The *project revenue* is the income earned by a business as a result of providing products or services to outsiders. Revenue comes from sales of merchandise to customers and from fees earned by services performed for clients or others.
2. The *project expenses* incurred are the cost of doing business to generate the revenues of that period. Some common expenses are the cost of the goods sold (labor, material, inventory, and supplies), depreciation, the cost of employees' salaries, the operating cost (such as the cost of renting a building and the cost of insurance coverage), and income taxes.

The business expenses listed above are all accounted for in a straightforward fashion on the income statement and balance sheet: the amount paid by the organization for each item would translate dollar for dollar into expenses in financial reports for the period. One additional category of expenses, the purchase of new assets, is treated by depreciating the total cost gradually over time. Because it plays a role in reducing taxable income, depreciation accounting is of special concern to a company.

Accounting Depreciation

The acquisition of fixed assets is an important activity for a business organization, whether the organization is starting up or acquiring new assets to remain competitive. The systematic allocation of the initial cost of an asset in parts over a time known as its depreciable life is what we mean by *accounting depreciation*. Because accounting depreciation is the standard of the business world, we sometimes refer to it more generally as *asset depreciation*.

The process of depreciating an asset requires that we make several preliminary determinations:

1. *What can be depreciable?* Depreciable property includes buildings, machinery, equipment, and vehicles. Inventories are not depreciable property because they are held primarily for sale to customers in the ordinary course of business. If an asset has no definite service life, the asset cannot be depreciated. For example, *you can never depreciate land*.
2. *What cost base should be used in asset depreciation?* The *cost base* of an asset represents the total cost that is claimed as an expense over an asset's life, that is, the sum of the annual depreciation expenses. The cost base generally includes the actual cost of the asset and all the other incidental expenses, such as freight, site preparation, and installation. This total cost, rather

than the cost of the asset only, must be the depreciation base charged as an expense over the asset's life.

3. *What is the asset's value at the end of its useful life?* The salvage value is an asset's value at the end of its life; it is the amount eventually recovered through sale, trade-in, or salvage. The eventual salvage value of an asset must be estimated when the depreciation schedule for the asset is established. If this estimate subsequently proves to be inaccurate, then an adjustment must be made.
4. *What is the depreciable life of the asset?* Historically, depreciation accounting included choosing a depreciable life that was based on the service life of an asset. Determining the service life of an asset, however, was often very difficult, and the uncertainty of these estimates often led to disputes between taxpayers and the IRS. To alleviate the problems, the IRS published guidelines on lives for categories of assets known as *Asset Depreciation Ranges*, or ADRs. These guidelines specified a range of lives for classes of assets based on historical data, and taxpayers were free to choose a depreciable life within the specified range for a given asset.
5. *What method of depreciation do we choose?* Companies generally calculate depreciation one way when figuring taxes and another way when reporting income (profit) to investors: (1) they use the straight-line method (or declining balance or sum-of-years' digits) for investors and (2) they use the fastest rate permitted by law (known as "modified accelerated cost recovery system [MACRS]") for tax purposes. Under the straight-line method, for an asset with a 5-year life which costs \$10,000 and has a \$1000 salvage value, the annual depreciation charge is $(\$10,000 - \$1000)/5 = \$1800$. For tax purposes, Congress created several classes of assets, each with a more or less arbitrarily prescribed life called a *recovery period* or *class life*. The depreciable base is not adjusted for salvage value, which is the estimated market value of the asset at the end of its useful life. [Table 17.4.1](#) describes what types of property fit into the different class life groups and the allowed depreciation percentages. Congress developed these recovery allowance percentages based on the declining balance method, with a switch to straight-line depreciation at some point in the asset's life. The MACRS recovery percentages as shown in [Table 17.4.1](#) also employ the half-year convention — that is, they assume that all assets are put into service at midyear and, hence, generate a half-year's depreciation.

Corporate Income Taxes

Corporate taxable income is defined as follows:

$$\begin{aligned} \text{taxable income} &= \text{gross income (revenues)} \\ &\quad - (\text{cost of goods sold} + \text{depreciation} + \text{operating expenses}) \end{aligned}$$

Once taxable income is calculated, income taxes are determined by

$$\text{income taxes} = (\text{tax rate}) \times (\text{taxable income})$$

The corporate tax rate structure for 1996 is relatively simple. There are four basic rate brackets (ranging from 15 to 35%) plus two surtax rates (5 and 3%) based on taxable incomes, and businesses with lower taxable incomes continue to be taxed at lower rates than those with higher taxable incomes.

Tax Treatment of Gains or Losses for Depreciable Assets

When a depreciable asset used in business is sold for an amount different from its book value, this gain or loss has an important effect on income taxes. An asset's book value at any given time is determined by

TABLE 17.4.1 MACRS Depreciation Schedules for Personal Properties with Half-Year Convention

Year	Class Depreciation Rate	Personal Property					
		3 200% DB	5 200% DB	7 200% DB	10 200% DB	15 150% DB	20 150% DB
1		33.33	20.00	14.29	10.00	5.00	3.750
2		44.45	32.00	24.49	18.00	9.50	7.219
3		14.81*	19.20	17.49	14.40	8.55	6.677
4		7.41	11.52*	12.49	11.52	7.70	6.177
5			11.52	8.93*	9.22	6.93	5.713
6			5.76	8.92	7.37	6.23	5.285
7				8.93	6.55*	5.90*	4.888
8				4.46	6.55	5.90	4.522
9					6.56	5.91	4.462*
10					6.55	5.90	4.461
11					3.28	5.91	4.462
12						5.90	4.461
13						5.91	4.462
14						5.90	4.461
15						5.91	4.462
16						2.95	4.461
17							4.462
18							4.461
19							4.462
20							4.461
21							2.231

Note: "*" denotes year to switch from declining balance to straight line.

Property Class	Applicable Property
3-year	Machine tools
5-year	Automobiles, light trucks, high-tech equipment
7-year	Manufacturing equipment, office furniture
10-year	Vessels, barges, tugs, railroad cars
15-year	Utility property (water, telephone)
20-year	Municipal sewers, electrical power plant

$$\text{book value} = \text{cost base} - \text{total amount of depreciation}$$

The gain or loss is found by

$$\text{gains (losses)} = \text{salvage value} - \text{book value}$$

where the salvage value represents the proceeds from the sale less any selling expense or removal cost.

These gains, commonly known as *depreciation recapture*, are taxed as ordinary income. In the unlikely event that if an asset is sold for an amount greater than its initial cost, the gains (salvage value – book value) are divided into two parts (ordinary gains and capital gains) for tax purposes:

$$\begin{aligned} \text{gains} &= \text{salvage value} - \text{book value} \\ &= \underbrace{(\text{salvage value} - \text{cost base})}_{\text{capital gains}} + \underbrace{(\text{cost base} - \text{book value})}_{\text{ordinary gains}} \end{aligned}$$

Capital gains are normally taxed at a different rate from that of ordinary gains.

After-Tax Cash Flow Analysis

In developing an after-tax flow, we are concerned only with those cash flows that result directly from the project. These cash flows, called *incremental cash flows*, represent the change in the firm’s total cash flows that occurs as a direct result of undertaking the project. There are several elements that contribute toward the project cash flows. In preparing the cash flow statement which shows sources and uses of cash in project undertaking, we may group them into three areas: (1) cash flow elements associated with operations, (2) cash flow elements associated with investment activities (capital expenditures), and (3) cash flow elements associated with project financing (such as borrowing). The main purpose of this grouping is to provide information about the operating, investing, and financing activities of a project.

- *Operating Activities:* In general, cash flows from operations include current sales revenue, cost of goods sold, operating expense, and income taxes. Cash flows from operations should generally reflect the cash effects of transactions entering into the determination of net income. The interest portion of a loan repayment is a deductible expense when determining net income, and it is included in the operating activities. Since we will usually look only at yearly flows, it is logical to express all cash flows on a yearly basis. We can determine the net cash flow from operations either (1) based on net income or (2) based on cash flow by computing income taxes directly. When we use net income as the starting point for cash flow determination, we should add any noncash expenses (mainly depreciation) back to net income to compute the net cash flow. Thus,

$$\begin{aligned} \text{net operating cash flow} &= \text{net income} + \text{noncash expenses} \\ &\quad (\text{depreciation}) \end{aligned}$$

Approach 1	Approach 2
Cash revenues (savings)	Cash revenues (savings)
–Cost of goods sold	–Cost of goods sold
–Depreciation	
–Operating expense	–Operating expense
–Interest expense	–Interest expense
Taxable income	
<u>–Income taxes</u>	<u>–Income taxes</u>
Net income + depreciation	Operating cash flow

In business practice, accountants usually prepare the cash flow statements based on the net income, namely, using Approach 1, whereas Approach 2 is commonly used in many traditional engineering economic texts. If you learn only Approach 2, it is more than likely that you need to be retrained to learn Approach 1 to communicate with the financing and accounting professionals within your organization. Therefore, we will use the income statement approach (Approach 1) whenever possible throughout this section.

- *Investing Activities:* Three types of investment flows are associated with buying a piece of equipment: the original investment, salvage value at the end of its useful life, and working capital investment* or recovery.** We will assume that our outflow for both capital investment and working capital investment is as if they take place in year 0. It is quite possible that both investments will not occur instantaneously, but rather over a few months as the project gets into gear; we could then use year 1 as an investment year. (Capital expenditures may occur over several years before a large investment project becomes fully operational. In this case, we should enter all expenditures as they occur.) For a small project, either method of timing these flows is satisfactory, because the numerical differences are likely to be insignificant.
- *Financing Activities:* Cash flows classified as financing activities include (1) the amount of borrowing and (2) repayment of principal. Recall that interest payments are tax deductible expenses so that they are usually classified as operating, not financing, activities.

Then, net cash flow for a given year is simply the sum of the net cash flows from these three activities.

Example 17.4.1 — Developing a Cash Flow Statement

A computerized machining center has been proposed for a small tool manufacturing company. If the new system costing \$125,000 is installed, it will generate annual revenues of \$100,000 and require \$20,000 in annual labor, \$12,000 in annual material expenses, and another \$8000 in annual overhead (power and utility) expenses. The automation facility would be classified as a 7-year MACRS property. The company expects to phase out the facility at the end of 5 years, at which time it will be sold for \$50,000. The machining center will require an investment of \$23,331 in working capital (mainly spare parts inventory), which will be recovered in full amount at the end of the project life. Assume that \$62,500 of the \$125,000 paid for the investment is obtained through a bank loan which is to be repaid in equal annual installments at 10% interest over 5 years. The remaining \$62,500 will be provided by equity (for example, from retained earnings). Find the year-by-year after-tax net cash flow for the project at a 40% marginal tax rate based on the net income (Approach 1), and determine the after-tax net present worth of the project at the company's MARR of 15%.

Discussion. We will use the business convention that no signs (positive or negative) are used in preparing the income statement, except in the situation where we have a negative taxable income or tax savings. In this situation we will use (). However, in preparing the cash flow statement, we will observe explicitly the sign convention: a positive sign indicating cash inflow, a negative sign or () indicating cash outflow.

Solution. Before presenting the cash flow table, we need some preliminary calculations. The following notes explain the essential items in [Table 17.4.2](#).

- *Depreciation Calculation:*
 1. If it is held for all 7 years, we depreciate a 7-year property in respective percentages of 14.29, 24.49, 17.49, 12.49, 8.93, 8.92, 8.93, and 4.46% (see [Table 17.4.1](#)).
 2. If the asset is sold at the end of the fifth tax year (during the recovery period), the applicable depreciation amounts would be \$17,863, \$30,613, \$21,863, \$15,613, and \$5581. Since the

* Normally, additional inventories are required to support a new operation and increased accounts receivable. These increases in current assets must be financed. On the other hand, accounts payable will also increase as a result of business expansion and this will reduce the net cash needed to finance inventories and accounts receivable. The difference is known as *net change in working capital*. If this difference is positive, an investment in working must be made.

** As the project approaches termination, inventories are sold off and not replaced, and account receivables are eventually collected. As this change occurs, the firm experiences an end-of-project cash flow (known as *working capital recovery*) that is equal to the net working capital investments that were made when the project was begun.

TABLE 17.4.2 Cash Flow Statement for the Automated Machining Center Project with Debt Financing (Example 17.4.1)

Income Statement						
	0	1	2	3	4	5
Revenues		\$100,000	\$100,000	\$100,000	\$100,000	\$100,000
Labor		20,000	20,000	20,000	20,000	20,000
Material		12,000	12,000	12,000	12,000	12,000
Overhead		8,000	8,000	8,000	8,000	8,000
Depreciation		17,863	30,613	21,863	15,613	5,581
Debt interest		6,250	5,226	4,100	2,861	1,499
Taxable income		\$35,887	\$24,161	\$34,037	\$41,526	\$52,920
Income taxes (40%)		<u>14,355</u>	<u>9,664</u>	<u>13,615</u>	<u>16,610</u>	<u>21,168</u>
Net income		\$21,532	\$14,497	\$20,433	\$24,916	\$31,752
Cash Flow Statement						
Operating activities						
Net income		21,532	14,497	20,422	24,916	31,752
Depreciation		17,863	30,613	21,863	15,613	5,581
Investment activities						
Investment	(125,000)					
Salvage						50,000
Gains tax						(6,613)
Working capital	(23,331)					23,331
Financing activities						
Borrowed funds	62,500					
Principal repayment		<u>(10,237)</u>	<u>(11,261)</u>	<u>(12,387)</u>	<u>(13,626)</u>	<u>(14,988)</u>
Net cash flow	\$(85,831)	\$29,158	\$33,849	\$29,898	\$26,903	\$89,063

asset is disposed of in the fifth tax year, the last year's depreciation, which would ordinarily be \$11,163, is halved due to the half-year convention.

- *Salvage Value and Gain Taxes:* In year 5, we must deal with two aspects of the asset's disposal — salvage value and gains (both ordinary as well as capital). We list the estimated salvage value as a positive cash flow. Taxable gains are calculated as follows:
 1. The total depreciation in years 1 to 5 is $\$17,863 + \$30,613 + \$21,863 + \$15,613 + \$5,581 = \$91,533$.
 2. The book value at the end of period 5 is the cost base minus the total depreciation, or $\$125,000 - \$91,533 = \$33,467$.
 3. The gains on the sale are the salvage value minus the book value, or $\$50,000 - \$33,467 = \$16,533$. (The salvage value is less than the cost base, so all the gain is ordinary.)
 4. The tax on the ordinary gains is $\$16,533 \times 40\% = \$6,613$. This is the amount that is placed in the table under "gains tax".
- *Interest and principal repayment:* We first need to compute the size of the annual installments:

$$\$62,500(A/P, 10\%, 5) = \$16,487$$

Next, we determine the repayment schedule of the loan by itemizing both the interest and the principal represented in each annual repayment as follows:

The interest payments are listed under the income statement and the principal payments are listed under the cash flow statement as shown in Table 17.4.2.

Year	Beginning Balance	Interest Payment	Principal Payment	Ending Balance
1	\$62,500	\$6,250	\$10,237	\$52,263
2	52,263	5,226	11,261	41,002
3	41,002	4,100	12,387	28,615
4	28,615	2,861	13,626	14,989
5	14,989	1,499	14,988	0

- *Investment Analysis:* Once we obtain the project's after-tax net cash flows, we can determine their equivalent present worth at the firm's interest rate of 15%. The present value equivalent of the after-tax cash flow series is

$$\begin{aligned}
 PW(15\%) &= -\$85,351 + \$29,158(P/F, 15\%, 1) + \dots + \$89,063(P/F, 15\%, 5) \\
 &= \$44,439
 \end{aligned}$$

This means that investing \$125,000 in this automated facility would bring in enough revenue to recover the initial investment and the cost of funds with a surplus of \$44,439.

Effects of Inflation on Project Cash Flows

We now introduce inflation into estimating project cash flows. We are especially interested in two elements of project cash flows — depreciation expenses and interest expenses — that are essentially immune to the effects of inflation. We also consider the complication of how to proceed when multiple price indexes have been used in generating various project cash flows.

Because depreciation expenses (as well as loan repayment) are calculated on some base-year purchase amount, they do not increase over time to keep pace with inflation. Thus, they lose some of their value to defer taxes as inflation drives up the general price level and hence taxable income. Similarly, salvage values of depreciable assets can increase with the general inflation rate and, because any gains on salvage values are taxable, they can result in increased taxes.

Example 17.4.2 — After-Tax Cash Flows under Inflation

A construction firm is offered a fixed-price contract for a 5-year period. The firm will be paid \$23,500 (actual dollars) per year for the contract period. In order to accept the contract, the firm must purchase equipment costing \$15,000 and requiring \$13,000 (constant dollars) per year to operate. The equipment qualifies for 5-year MACRS depreciation and is expected to have a salvage value of \$1000 at the end of 5 years. Use a tax rate of 40% and an inflation-free interest rate of 20%. If the general inflation rate (\bar{f}) is expected to average 5% over the next 5 years, salvage value at the annual rate of 5%, and operating expenses at 8% per year for the project duration, should the contractor accept the contract?

Solution. The analysis of this situation should explicitly consider inflation, since the contractor is being offered a fixed-price contract and cannot increase the fee to compensate for increased costs. In this problem, as is common practice, we assume that all estimated costs are expressed in today's dollars, and actual costs will be increased due to inflation.

The Excel spreadsheet analysis is shown in [Table 17.4.3](#), where the first five rows are designated as the input fields. Then column C in both the income statement and the cash flow statement is reserved for entering the specific inflation rate for the item listed in that row. Since the O&M costs are responsive to inflation, we can automate the cell entries by using the following cell formulas:

TABLE 17.4.3 Excel’s Spreadsheet Application to “What If” Questions — What General Inflation Rate Does the Project Break Even? (Example 17.15)

	A	B	C	D	E	F	G	H	I	
1										
2		INPUT:	O&M Cost	\$ 13,000		General Inflation Rate	5.00%			
3			Salvage	\$ 1,000		Inflation-Free Interest	20.00%			
4			Contract \$	\$ 23,000		Market Interest Rate	26.00%			
5			Investment	\$ 15,000		Income Tax Rate	40.00%			
6										
7			Inflation Rate	0	1	2	3	4	5	
8		Income Statement								
9										
10		Revenues			\$ 23,500	\$ 23,500	\$ 23,500	\$ 23,500	\$ 23,500	
11		Expenses:								
12		O&M	8%		14,040	15,163	16,376	17,686	19,101	
13		Depreciation			3,000	4,800	2,880	1,728	864	
15		Taxable Income			\$ 6,460	\$ 3,537	\$ 4,244	\$ 4,066	\$ 3,535	
16		Income Taxes (40%)			2,584	1,415	1,697	1,634	1,414	
18		Net income			\$ 3,876	\$ 2,122	\$ 2,546	\$ 2,451	\$ 2,121	
19										
20		Cash Flow Statement								
21										
22		Operating Activities:								
23		Net Income			3,876	2,122	2,546	2,451	2,121	
24		Depreciation			3,000	4,800	2,880	1,728	864	
25		Investment Activities:								
26		Investment			(15,000)					
27		Salvage	5%						1,276	
28		Gains Tax							181	
30		Net Cash Flow			\$ (15,000)	\$ 6,876	\$ 6,922	\$ 5,426	\$ 4,179	\$ 4,442
31		(in actual dollars)								
32		Net Cash Flow			\$ (15,000)	\$6,549	\$6,279	\$4,687	\$3,438	\$3,480
33		(in constant dollars)								
34		Equ. Present Worth			\$ (15,000)	\$5,457	\$4,360	\$2,713	\$1,658	\$1,399
36		Net Present Worth			\$587					
37		Internal Rate of Return			22.03%					

$$\text{cell E12} = \text{\$D\$2} * (1 + \text{\$C\$12})$$

$$\text{cell F12} = \text{E12} * (1 + \text{\$C\$12})$$

⋮ ⋮

$$\text{cell I12} = \text{H12} * (1 + \text{\$C\$12})$$

Note that the salvage value in cell I27 has been increased at 5%. (When no inflation rate is given for a specific cost category, it is normally assumed that the general inflation rate holds.) Gains taxes on disposal are based on the difference between book value and salvage value, in this case, \$1276 – \$1728 = (\$452) or loss of \$452. This results in a tax savings of (\$452) (0.40) = \$181, as shown in cell I28.

In row 12, operating expenses have been increased at 8%. As explained in the section “Loss of Purchasing Power,” actual-dollar cash flows are converted to constant-dollar flows by “deflating” at the general inflation rate. The IRR is calculated at 22.03%, based on the constant-dollar cash flows. This is compared to the MARR of 20%, the inflation-free interest rate, indicating that this is an acceptable contract as long as the 5 and 8% escalation rates are correct.

Since no one can predict inflation rates in any precise manner, it is always wise to investigate the effects of changes in these rates. This is very easy to do with a spreadsheet. For example, we can easily determine the value of the general inflation rate at which this project exactly earns 20%. The value of \hat{f} was adjusted manually until the IRR was exactly 20%. This would not be a good project if \hat{f} were greater than 6.77% (cell H2). (Note that at the exact value of \hat{f} (6.771%), the NPW in cell C36 will be zero or its rate of return would be 20%.) Similarly, you can vary the O&M cost and salvage value to see how the project's profitability changes.

17.5 Sensitivity and Risk Analysis

In previous sections, the cash flows from projects were assumed to be known with complete certainty, and our analysis was concerned with measuring the economic worth of projects and selecting the best investment projects. Although these results can provide reasonable decision bases for many investment situations, we should certainly consider the more usual situation where forecasts of cash flows are subject to some degree of uncertainty. In this situation, management rarely has precise expectations regarding the future cash flows to be derived from a particular project. In fact, the best that the firm can reasonably expect to do is to estimate the range of possible future costs and benefits and the relative chances of achieving a certain return on the investment. We can use the term risk in describing an investment project whose cash flow is not known in advance with absolute certainty, but for which an array of alternative outcomes and their probabilities (odds) are known. We will also use the term project risk to refer to the variability in a project's NPW. A greater project risk means that there is a greater variability in the project's NPW, or simply saying that *risk is the potential for loss*. We may begin analyzing project risk by first determining the uncertainty inherent in a project's cash flows. We can consider risk in a number of ways ranging from making informal judgments to calculating complex economic and statistical analyses. In this section, we will introduce three methods of describing project risk: (1) sensitivity analysis, (2) scenario analysis, and (3) risk analysis. We shall explain each method with a single example (Boston Metal Company).

Project Risk

The decision to make a major capital investment such as the introduction of a new product requires cash flow information over the life of the project. The profitability estimate of the investment depends on cash flow estimations, which are generally uncertain. Many cash flow elements (such as demand) are subject to substantial uncertainty. The common approach is to make single-number "best estimates" for each of the uncertain factors and then to calculate measures of profitability such as NPW or rate of return for the project. This approach has two drawbacks:

1. There is no guarantee that the "best estimates" will ever match actual values.
2. There is no way to measure the risk associated with the investment or the project risk. In particular, the manager has no way of determining either the probability that the project will lose money or the probability that it will generate very large profits.

Because cash flows can be so difficult to estimate accurately, project managers frequently consider a range of possible values for cash flow elements. If there is a range of possible values for individual cash flows, it follows that there is a range of possible values for the NPW of a given project. Clearly, the analyst will want to try to gauge the probability and reliability of individual cash flows occurring and, consequently, the level of certainty about achieving the overall project worth.

Sensitivity Analysis

One way to glean a sense of the possible outcomes of an investment is to perform a sensitivity analysis. This analysis determines the effect on NPW of variations in the input variables (such as revenues,

operating cost, and salvage value) used to estimate after-tax cash flows. A *sensitivity analysis* reveals how much the NPW will change in response to a given change in an input variable. In a calculation of cash flows, some items have a greater influence on the final result than others. In some problems, we may easily identify the most significant item. For example, the estimate of sales volume is often a major factor in a problem in which the quantity sold varies among the alternatives. In other problems, we may want to locate the items that have an important influence on the final results so that they can be subjected to special scrutiny.

Sensitivity analysis is sometimes called “what-if” analysis because it answers questions such as: What if incremental sales are only 1000 units, rather than 2000 units? Then what will the NPW be? Sensitivity analysis begins with a base-case situation, which is developed using the most-likely values for each input. Then we change the specific variable of interest by several specific percentages above and below the most-likely value, holding other variables constant. Next, we calculate a new NPW for each of these values. A convenient and useful way to present the results of a sensitivity analysis is to plot *sensitivity graphs*. The slopes of the lines show how sensitive the NPW is to changes in each of the inputs: the steeper the slope, the more sensitive the NPW is to a change in the particular variable. It identifies the crucial variables that affect the final outcome most. We will use Example 17.5.1 to illustrate the concept of sensitivity analysis.

Example 17.5.1 — Sensitivity Analysis

Boston Metal Company (BMC), a small manufacturer of fabricated metal parts, must decide whether to enter the competition to be the supplier of transmission housings for Gulf Electric. Gulf Electric has its own in-house manufacturing facility to produce transmission housings, but it has almost reached its maximum production capacity. Therefore, Gulf is looking for an outside supplier. To compete, the firm must design a new fixture for the production process and purchase a new forge. The new forge would cost \$125,000, including tooling costs for the transmission housings. If BMC gets the order, it may be able to sell as many as 2000 units per year to Gulf Electric for \$50 each, and the variable production costs,* such as direct labor and direct material costs, will be \$15 per unit. The increase in fixed costs** other than depreciation will amount to \$10,000 per year. The firm expects that the proposed transmission-housings project would have about a 5-year product life. The firm also estimates that the amount ordered by Gulf Electric for the first year will be ordered in each of the subsequent 4 years. (Due to the nature of contracted production, the annual demand and unit price would remain the same over the project after the contract is signed.) The initial investment can be depreciated on a MACRS basis over the 7-year period, and the marginal income tax rate is expected to remain at 40%. At the end of 5 years, the forge machine is expected to retain a market value of about 35% of the original investment. Based on this information, the engineering and marketing staffs have prepared the cash flow forecasts shown in [Table 17.5.1](#). Since NPW is positive (\$40,169) at the 15% opportunity cost of capital (MARR), the project appears to be worth undertaking.

However, BMC’s managers are uneasy about this project because there are too many uncertain elements that have not been considered in the analysis. If decided, BMC must make the investment in the forging machine to provide some samples with Gulf Electric as a part of the bidding process. If Gulf Electric does not like BMC’s sample, BMC stands to lose the entire investment in the forging machine. On the other hand, if Gulf likes BMC’s sample but it is overpriced, BMC would be under pressure to bring the price in line with competing firms. There is even a possibility that BMC would get a smaller order as Gulf may utilize their overtime capacity to produce some extra units. They also are not certain about the variable and fixed cost figures. Recognizing these uncertainties, the managers want to assess the various potential future outcomes before making a final decision. (a) Perform a sensitivity analysis to each variable and (b) develop a sensitivity graph.

* Expenses that change in direct proportion to the change in volume of sales or production.

** Expenses that do not vary as the volume of sales or production changes. For example, property taxes, insurance, depreciation, and rent are usually fixed expenses.

TABLE 17.5.1 A Typical Excel Worksheet Design to Perform Sensitivity Analyses (Example 17.5.1)

	A	B	C	D	E	F	G	H
1								
2		Input Data (Base):			Sensitivity Analysis:			
3		Unit Price (\$)	50.00					
4		Demand	2000		Category		% Change	
5		Var. cost (\$/unit)	15.00		Unit price		0%	
6		Fixed cost (\$)	10000		Demand		0%	
7		Salvage (\$)	40000		Var. cost (unit)		0%	
8		Tax rate (%)	40%		Fixed cost		0%	
9		MARR (%)	15%		Salvage		0%	
10								
11					Output (NPW)		\$40,169	
12								
13			0	1	2	3	4	5
14		Income Statement						
15								
16		Revenues:						
17		Unit Price	\$ 50.00	\$ 50.00	\$ 50.00	\$ 50.00	\$ 50.00	\$ 50.00
18		Demand (units)	2000	2000	2000	2000	2000	2000
19		Sales Revenue	\$ 100,000	\$ 100,000	\$ 100,000	\$ 100,000	\$ 100,000	\$ 100,000
20		Expenses:						
21		Unit Variable Cost	\$ 15	\$ 15	\$ 15	\$ 15	\$ 15	\$ 15
22		Variable Cost	30,000	30,000	30,000	30,000	30,000	30,000
23		Fixed Cost	10,000	10,000	10,000	10,000	10,000	10,000
24		Depreciation	17,863	30,613	21,863	15,613	5,581	
26		Taxable Income	\$ 42,137	\$ 29,387	\$ 38,137	\$ 44,387	\$ 54,419	
27		Income Taxes (40%)	16,855	11,755	15,255	17,755	21,768	
29		Net Income	\$ 25,282	\$ 17,632	\$ 22,882	\$ 26,632	\$ 32,651	
30								
31		Cash Flow Statement						
32								
33		Operating Activities:						
34		Net Income	25,282	17,632	22,882	26,632	32,651	
35		Depreciation	17,863	30,613	21,863	15,613	5,581	
36		Investment Activities:						
37		Investment	(125,000)					
38		Salvage					40,000	
39		Gains Tax					(2,613)	
41		Net Cash Flow	\$ (125,000)	\$ 43,145	\$ 48,245	\$ 44,745	\$ 42,245	\$ 75,619

Discussion. Table 17.5.1 shows BMC’s expected cash flows — but there is no guarantee that they will indeed materialize. BMC is not particularly confident in its revenue forecasts. The managers think that if competing firms enter the market, BMC will lose a substantial portion of the projected revenues. Before undertaking the project described, the company wants to identify the key variables that determine whether the project will succeed or fail. The marketing department has estimated revenue as follows:

$$\begin{aligned} \text{annual revenue} &= (\text{product demand})(\text{unit price}) \\ &= (2000)(\$50) = \$100,000 \end{aligned}$$

The engineering department has estimated variable costs such as labor and material per unit at \$15. Since the projected sales volume is 2000 units per years, the total variable cost is \$30,000.

Having defined the unit sales, unit price, unit variable cost, and fixed cost, we conduct a sensitivity analysis with respect to these key input variables. This is done by varying each of the estimates by a

given percentage and determining what effect the variation in that item has on the final results. If the effect is large, the result is sensitive to that item. Our objective is to locate the most sensitive item(s).

Solution.

- (a) *Sensitivity analysis:* We begin the sensitivity analysis with a “base-case” situation, which reflects the best estimate (expected value) for each input variable. In developing [Table 17.5.2](#), we changed a given variable by 20% in 5% increments, above and below the base-case value, and calculated new NPWs, holding other variables constant. The values for both sales and operating costs were the expected, or base-case, values, and the resulting \$40,169 is called the base-case NPW. Now we ask a series of “what-if” questions: What if sales are 20% below the expected level? What if operating costs rise? What if the unit price drops from \$50 to \$45? [Table 17.5.2](#) summarizes the results of varying the values of the key input variables.
- (b) *Sensitivity graph:* [Figure 17.5.1](#) shows the transmission project’s sensitivity graphs for six of the key input variables. The base-case NPW is plotted on the ordinate of the graph at the value 1.0 on the abscissa. Next, the value of product demand is reduced to 0.95 of its base-case value, and the NPW is recomputed with all other variables held at their base-case value. We repeat the process by either decreasing or increasing the relative deviation from the base case. The lines for the variable unit price, variable unit cost, fixed costs, and salvage value are obtained in the same manner. In [Figure 17.5.1](#), we see that the project’s NPW is very sensitive to changes in product demand and unit price, is fairly sensitive to changes in the variable costs, and is relatively insensitive to changes in the fixed cost and the salvage value.

Graphic displays such as those in [Figure 17.5.1](#) provide a useful means to communicate the relative sensitivities of the different variables on the corresponding NPW value. However, the sensitivity graph does not explain any variable interactions among the variables or the likelihood of realizing any specific deviation from the base case. Certainly, it is conceivable that an answer might not be very sensitive to changes in either of the two items, but very sensitive to combined changes in them.

Scenario Analysis

Although sensitivity analysis is useful, it does have limitations. It is often difficult to specify precisely the relationship between a particular variable and the NPW. The relationship is further complicated by interdependencies among the variables. Holding operating costs constant while varying unit sales may ease the analysis, but in reality, operating costs do not behave in that manner. Yet it may complicate the analysis too much to permit movement in more than one variable at a time. A scenario analysis is a technique that does consider the sensitivity of NPW both to changes in key variables and to the range of likely variable values. For example, the decision-maker may consider two extreme cases, a “worst-case” scenario (low unit sales, low unit price, high variable cost per unit, high fixed cost, and so on) and a “best-case” scenario. The NPWs under the worst and best conditions are then calculated and compared to the expected, or base-case, NPW. [Example 17.5.2](#) will illustrate a plausible scenario analysis for the BMC’s transmission-housings project.

Example 17.5.2 — Scenario Analysis

Consider again BMC’s transmission-housing project in [Example 17.5.2](#). Assume that the company’s managers are fairly confident of their estimates of all the projects’ cash flow variables except that for unit sales. Further, assume that they regard a decline in unit sales to below 1600 or a rise above 2400 as extremely unlikely. Thus, decremental annual sales of 400 units define the lower bound, or the worst-case scenario, whereas incremental annual sales of 400 units define the upper bound, or the best-case scenario. (Remember that the most-likely value was 2000 in annual unit sales.) Discuss the worst- and best-case scenarios, assuming that the unit sales for all 5 years would be equal.

TABLE 17.5.2 Sensitivity Analysis for Six Key Input Variables (Example 17.5.1)

Deviation	Base									
	-20%	-15%	-10%	-5%	0%	5%	10%	15%	20%	
Unit price	\$ (57)	\$ 9,999	\$ 20,055	\$ 30,111	\$ 40,169	\$ 50,225	\$ 60,281	\$ 70,337	\$ 80,393	
Demand	\$ 12,010	\$ 19,049	\$ 26,088	\$ 33,130	\$ 40,169	\$ 47,208	\$ 54,247	\$ 61,286	\$ 68,325	
Variable cost	\$ 52,236	\$ 49,219	\$ 46,202	\$ 43,186	\$ 40,169	\$ 37,152	\$ 34,135	\$ 31,118	\$ 28,101	
Fixed cost	\$ 44,191	\$ 43,185	\$ 42,179	\$ 41,175	\$ 40,169	\$ 39,163	\$ 38,157	\$ 37,151	\$ 36,145	
Salvage value	\$ 37,782	\$ 38,378	\$ 38,974	\$ 39,573	\$ 40,169	\$ 40,765	\$ 41,361	\$ 41,957	\$ 42,553	
MARR	\$ 53,588	\$ 50,075	\$ 46,670	\$ 43,369	\$ 40,169	\$ 37,064	\$ 34,051	\$ 32,128	\$ 28,289	

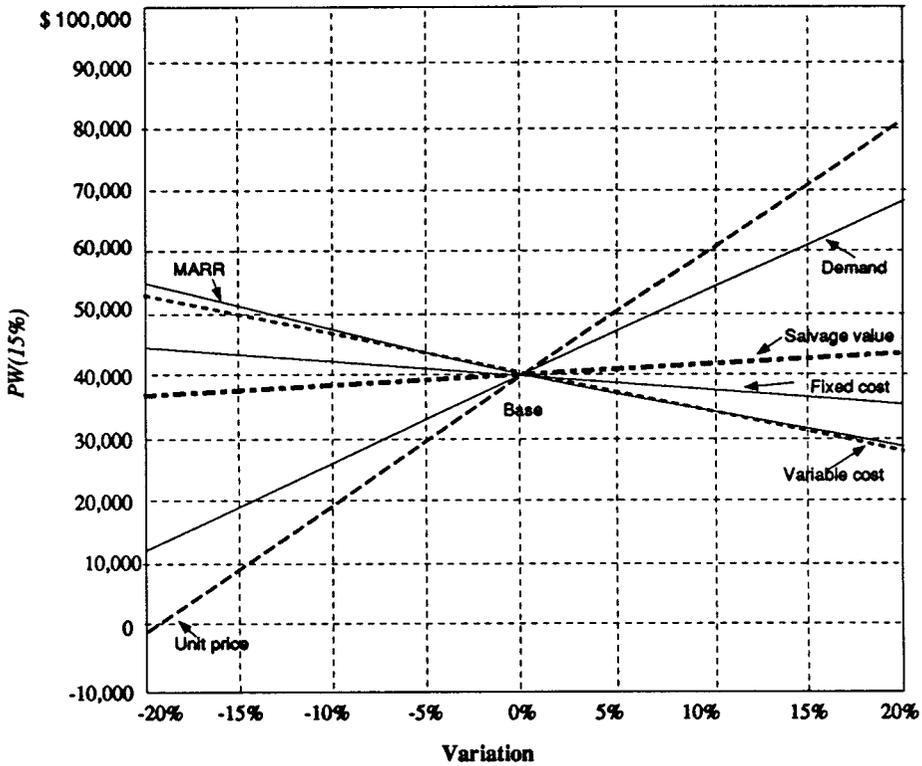


FIGURE 17.5.1 Sensitivity graph — BMC’s transmission-housings project (Example 17.5.1).

Discussion. To carry out the scenario analysis, we ask the marketing and engineering staffs to give optimistic (best-case) and pessimistic (worst-case) estimates for the key variables. Then we use the worst-case variable values to obtain the worst-case NPW and the best-case variable values to obtain the best-case NPW. Table 17.5.3 summarizes the results of our analysis. We see that the base case produces a positive NPW, the worst case produces a negative NPW, and the best case produces a large positive NPW.

TABLE 17.5.3 Scenario Analysis for BMC (Example 17.5.2)

Variable Considered	Worst-Case Scenario	Most-Likely-Case Scenario	Best-Case Scenario
Unit demand	1,600	2,000	2,400
Unit price (\$)	48	50	53
Variable cost (\$)	17	15	12
Fixed cost (\$)	11,000	10,000	8,000
Salvage value (\$)	30,000	40,000	50,000
PW (15%)	-\$5,856	\$40,169	\$104,295

By just looking at the results in Table 17.5.3, it is not easy to interpret scenario analysis or to make a decision based on it. For example, we can say that there is a chance of losing money on the project, but we do not yet have a specific probability for this possibility. Clearly, we need estimates of the probabilities of occurrence of the worst case, the best case, the base case (most likely), and all the other possibilities. This need leads us directly to the next step, developing a probability distribution. If we can predict the effects on the NPW of variations in the parameters, why should we not assign a probability distribution to the possible outcomes of each parameter and combine these distributions in some way

to produce a probability distribution for the possible outcomes of the NPW? We shall consider this issue in the next section.

Risk Analysis

Quantitative statements about risk are given as numerical probabilities, or as values for likelihoods (odds) of occurrence. Probabilities are given as decimal fractions in the interval 0.0 to 1.0. An event or outcome that is certain to occur has a probability of 1.0. As the probability of an event approaches 0, the event becomes increasingly less likely to occur. The assignment of probabilities to the various outcomes of an investment project is generally called *risk analysis*. In this section, we shall assume that the analyst has available the probabilities (likelihoods) of future events from either previous experience in a similar project or a market survey. The use of probability information can provide management with a range of possible outcomes and the likelihood of achieving different goals under each investment alternative.

Procedure for Developing an NPW Distribution

To develop the NPW distribution, we may follow these steps:

1. Identify all cash flows elements that are random variables. (A *random variable*) is a variable that can have more than one possible value.)
2. Express the NPW (or cash flow series) as a function of random variables.
3. Determine the probability distribution for each random variable.
4. Determine the joint events and their probabilities.
5. Evaluate the NPW equation at these joint events.
6. Order the NPW values in increasing order of NPW.

These steps can best be illustrated by Example 17.5.3.

Example 17.5.3 — Developing an NPW Probability Distribution

Consider the BMC's transmission-housing project. If the unit sales (X) and unit price (Y) were to vary with the following probabilities, determine the NPW probability distribution. Here we assume the situation where both random variables are independent. In other words, observing a typical outcome for random variable X does not have any influence on predicting the outcome for random variable Y .

Demand (X)	Probability	Unit Price (Y)	Probability
1600	0.20	\$48	0.30
2000	0.60	\$50	0.50
2400	0.20	\$53	0.20

Discussion. If the product demand X and the unit price Y are independent random variables, the PW (15%) will also be a random variable. To determine the NPW distribution, we need to consider all the combinations of possible outcomes. The first possibility is the event where $x = 1600$ and $y = \$48$. With these values as input in [Table 17.5.1](#), we compute the resulting NPW outcome to be \$5574. Since X and Y are considered to be independent random variables, the probability of this joint event is

$$\begin{aligned}
 P(x = 1600, y = \$48) &= P(x = 1600)P(y = \$48) \\
 &= (0.20)(0.30) \\
 &= 0.06
 \end{aligned}$$

TABLE 17.5.4 The NPW Probability Distribution with Independent Random Variables (Example 17.5.3)

Event No.	x	y	P(x,y)	Cumulative Joint Probability	NPW
1	1600	\$48.00	0.06	0.06	\$5,574
2	1600	\$50.00	0.10	0.16	\$12,010
3	1600	\$53.00	0.04	0.20	\$21,664
4	2000	\$48.00	0.18	0.38	\$32,123
5	2000	\$50.00	0.30	0.68	\$40,168
6	2000	\$53.00	0.12	0.80	\$52,236
7	2400	\$48.00	0.06	0.86	\$58,672
8	2400	\$50.00	0.10	0.96	\$68,326
9	2400	\$53.00	0.04	1.00	\$82,808

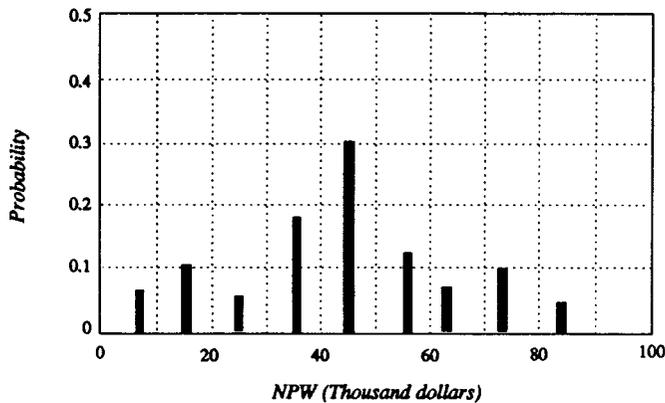


FIGURE 17.5.2 NPW probability distributions: when X and Y are independent (Example 17.5.3).

There are eight other possible joint outcomes. Substituting these pairs of values in Table 17.5.1, we obtain the NPWs and their joint probabilities in Table 17.5.4 and its NPW distribution as depicted in Figure 17.5.2.

Solution. The NPW probability distribution in Table 17.5.4 indicates that the project’s NPW varies between \$5574 and \$82,808, but that there is no loss under any of the circumstances examined. From the cumulative distribution, we further observe that there is a 0.38 probability that the project would realize an NPW less than that forecast for the base-case situation (\$40,168). On the other hand, there is a 0.32 probability that the NPW will be greater than this value. Certainly, the probability distribution provides much more information on the likelihood of each possible event, as compared with the scenario analysis presented in Table 17.5.3.

Expected Value and Variance

We have developed a probability distribution for the NPW by considering the random cash flows. As we have observed, the probability distribution helps us to see what the data imply in terms of the risk of the project. With the NPW distribution defined, we can further summarize the probabilistic information — the mean and the variance.

The expected value (also called the *mean*) is a weighted average value of the random variable (μ) where the weighting factors are the probabilities of occurrence. (See Table 17.5.5.) The expected value of a distribution tells us important information about the “average” or expected value of a random variable such as the NPW, but it does not tell us anything about the variability on either side of the

TABLE 17.5.5 Calculation of the Mean and Variance of NPW Distribution (Example 17.5.3)

Event No.	x	y	P(x,y)	Cumulative Joint		NPW	Weighted NPW
				Probability			
1	1600	\$48.00	0.06	0.06		\$5,574	\$334
2	1600	\$50.00	0.10	0.16		\$12,010	\$1,201
3	1600	\$53.00	0.04	0.20		\$21,664	\$867
4	2000	\$48.00	0.18	0.38		\$32,123	\$5,782
5	2000	\$50.00	0.30	0.68		\$40,168	\$12,050
6	2000	\$53.00	0.12	0.80		\$52,236	\$6,268
7	2400	\$48.00	0.06	0.86		\$58,672	\$3,520
8	2400	\$50.00	0.10	0.96		\$68,326	\$6,833
9	2400	\$53.00	0.04	1.00		\$82,808	\$3,312
E(NPW) = \$40,168							
Event No.	x	y	P(x,y)	NPW	(NPW – E[NPW]) ²	Weighted (NPW – E[NPW]) ²	
1	1600	\$48.00	0.06	\$5,574	1,196,769,744	71,806,185	
2	1600	\$50.00	0.10	\$12,010	792,884,227	79,288,423	
3	1600	\$53.00	0.04	\$21,664	342,396,536	13,695,861	
4	2000	\$48.00	0.18	\$32,123	64,725,243	11,650,544	
5	2000	\$50.00	0.30	\$40,168	0	0	
6	2000	\$53.00	0.12	\$52,236	145,631,797	17,475,816	
7	2400	\$48.00	0.06	\$58,672	342,396,536	20,543,792	
8	2400	\$50.00	0.10	\$68,326	792,884,227	79,288,423	
9	2400	\$53.00	0.04	\$82,808	1,818,132,077	72,725,283	
Var[PW(15%)] = 366,474,326							
Standard Deviation = \$19,144							

expected value. Will the range of possible values of the random variable be very small, with all the values located at or near the expected value?

Another measure that we need in analyzing probabilistic situations is a measure of the risk due to the variability of the outcomes. There are several measures of the variation of a set of numbers that are used in statistical analysis — the range and the variance (or standard deviation), among others. The variance and the standard deviation are used most commonly in the analysis of risk situations. We will use $\text{Var}[X]$ or σ_x^2 to denote the variance, and σ_x to denote the standard deviation of random variable X. (If there is only one random variable in an analysis, we normally omit the subscript.) The variance tells us the degree of spread, or dispersion, of the distribution on either side of the mean value. As the variance increases, the spread of the distribution increases; the smaller the variance, the narrower the spread about the expected value.

To determine the variance, we first calculate the deviation of each possible outcome x_j from the expected value $(x_j - \mu)$, then raise the result to the second power and multiply it by the probability of x_j occurring (that is, p_j). The summation of all these products serves as a measure of the distribution's variability. To be most useful, any measure of risk should have a definite value (unit). One such measure is the standard deviation. To calculate the standard deviation, we take the positive square root of $\text{Var}[X]$, which is measured in the same units as is X. The standard deviation is a probability-weighted deviation (more precisely, square root of sum of squared deviations) from the expected value. Thus, it gives us an idea of how far above or below the expected value the actual value is likely to be. For most probability distributions, the actual value will be observed within the $\pm 3\sigma$ range. In our BMC project, we obtain the variance of the NPW distribution, assuming independence between X and Y, as shown in Table 17.5.5.

Decision Rule

Once the expected value has been located from the NPW distribution, it can be used to make an accept-reject decision, in much the same way that a single NPW is used when a single possible outcome is considered for an investment project. The decision rule is called the *expected value criterion*, and using it we may accept a single project if its expected NPW value is positive. In the case of mutually exclusive alternatives, we select the one with the highest expected NPW. The use of expected NPW has an advantage over the use of a point estimate, such as the likely value, because it includes all the possible cash flow events and their probabilities.

The justification for the use of the expected value criterion is based on the *law of large numbers*, which states that if many repetitions of an experiment are performed, the average outcome will tend toward the expected value. This justification may seem to negate the usefulness of the expected value criterion in economic analysis, since most often in project evaluation we are concerned with a single, nonrepeatable “experiment” — that is, investment alternative. However, if a firm adopts the expected value criterion as a standard decision rule for *all* its investment alternatives, over the long term the law of large numbers predicts that accepted projects will tend to meet their expected values. Individual projects may succeed or fail, but the average project result will tend to meet the firm’s standard for economic success.

The expected-value criterion is simple and straightforward to use, but it fails to reflect the variability of investment outcome. Certainly, we can enrich our decision by incorporating the variability information along with the expected value. Since the variance represents the dispersion of the distribution, it is desirable to minimize it. In other words, the smaller the variance, the less the variability (the potential for loss) associated with the NPW. Therefore, when we compare the mutually exclusive projects, we may select the alternative with the smaller variance if its expected value is the same as, or larger than, those of other alternatives. In cases where there are no clear-cut preferences, the ultimate choice will depend on the decision-maker’s trade-offs — how much he or she is willing to take the variability to achieve a higher expected value. In other words, the challenge is to decide what level of risk you are willing to accept and then, having decided on your risk tolerance, to understand the implications of that choice.

17.6 Design Economics

Engineers frequently have to come up with a minimum-cost solution when they have two or more cost components that are affected differently by the same design element. That is, for a single design variable, some costs increase while others decrease. Another valuable extension of the AE analysis in the section on “Annual Equivalent Method,” is that these cost components be expressed in equivalent annual form so that we can identify which cost component we need to control to obtain the minimum cost solution to various engineering design problems.

Capital Costs vs. Operating Costs

The AE method is sometimes called the annual equivalent cost method when only costs are involved. In this case, there are two kinds of costs that revenues must cover: *operating costs* and *capital costs*. Operating costs are incurred by the operation of physical plant or equipment to provide service; they include such items as labor and raw materials. Capital costs are incurred by purchasing the assets used in production and service. Normally, capital costs are nonrecurring (that is, one-time costs), whereas operating costs recur as long as the asset is owned.

Because operating costs recur over the life of a project, they tend to be estimated on an annual basis anyway, so for the purposes of an annual equivalent cost analysis, they require no special calculation on our part. However, because capital costs tend to be one-time costs in conducting an annual equivalent

cost analysis, we must translate this one-time cost into its annual equivalent over the life of the project. The annual equivalent of a capital cost is given a special name: *capital recovery cost*, designated $CR(i)$.

There are two general monetary transactions associated with the purchase and eventual retirement of a capital asset, its initial cost (I) and its salvage value (S). Taking into account these sums, we calculate the capital recovery cost as

$$CR(i) = I(A/P, i, N) - S(A/F, i, N) \quad (17.6.1)$$

Recalling the algebraic relationships between factors in Table 17.1.1, notice that the $(A/P, i, N)$ factor can be expressed as

$$(A/P, i, N) = (A/F, i, N) + i$$

Then, we may rewrite the $CR(i)$ as

$$\begin{aligned} CR(i) &= I(A/P, i, N) - S[(A/P, i, N) - i] \\ &= (I - S)(A/P, i, N) + iS \end{aligned} \quad (17.6.2)$$

Minimum-Cost Function

When the equivalent annual total cost of a design variable is a function of increasing (O&M costs) and decreasing cost components (capital costs), we usually can find the optimal value that will minimize its cost.

$$AE(i) = a + bx + \frac{c}{x} \quad (17.6.3)$$

where x is a common design variable, and a , b , and c are constants.

To find the value of the common design variable that minimizes the $AE(i)$, we need to take the first derivative, equate the result to zero, and solve for x .

$$\begin{aligned} \frac{dAE(i)}{dx} &= b - \frac{c}{x^2} \\ &= 0 \end{aligned} \quad (17.6.4)$$

$$x^* = \sqrt{\frac{c}{b}}$$

The value x^* is the minimum cost point for the design alternative. To illustrate the optimization concept, we will consider selecting an optimal pipe size.

Example 17.6.1 — Economical Pipe Size

As a result of the conflict in the Persian Gulf, Kuwait is studying the feasibility of running a steel pipeline across the Arabian Peninsula to the Red Sea. The pipeline will be designed to handle 3 million barrels of crude oil per day at optimum conditions. The length of the line will be 600 miles. Calculate the optimum pipeline diameter that will be used for 20 years for the following data at $i = 10\%$:

- Pumping power = $1.333Q\Delta P/1,980,000$ horsepower
- Q = volume flow rate, cubic ft/hr

- $\Delta P = 128Q\mu L/g\pi D^4$, pressure drop lb/sq ft
- L = pipe length, ft
- D = inside pipe diameter, ft
- $t = 0.01 D$, pipeline wall thickness, ft
- $\mu = 8500$ lb/hr ft, oil viscosity
- $g = 32.2 \times 12,960,000$ ft/hr²
- Power cost, \$0.015 per horsepower hour
- Oil cost, \$18 per barrel
- Pipeline cost, \$1.00 per pound of steel
- Pump and motor costs, \$195 per horsepower

The salvage value of the steel after 20 years is assumed to be zero considering the cost of removal. (See Figure 17.6.1 for relationship between D and t .)

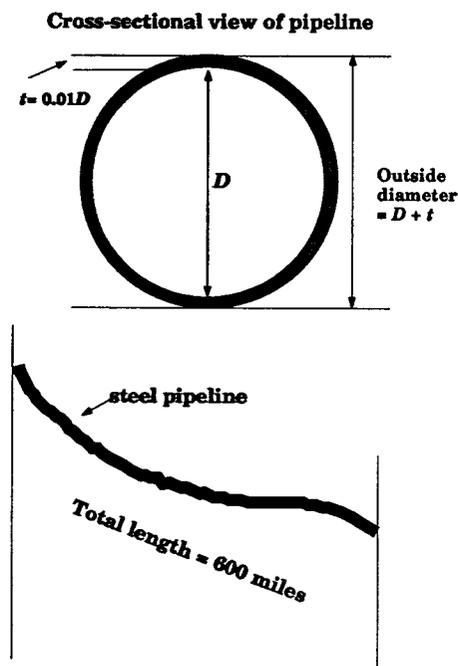


FIGURE 17.6.1 Designing economical pipe size to handle 3 million barrels of crude oil per day (Example 17.6.1).

Discussion. In general, when progressively larger-size pipe is used to carry a given fluid at a given volume flow rate, the energy required to move the fluid will progressively decrease. However, as we increase the pipe size, the cost to construct the pipe will increase. In practice, to obtain the best pipe size for a particular situation, you may choose a reasonable, but small, starting size. Compute the energy cost of pumping fluid through this size and the total construction cost. Compare the difference in energy cost with the difference in construction cost. When the savings in energy cost exceed the added construction cost, you may repeat the process with progressively larger pipe sizes until the added construction cost exceeds the savings in energy cost. As soon as this happens the best pipe size to use in the particular application is identified. However, we can simplify this search process by using the minimum cost concept as explained through Equations (17.6.3) and (17.6.4).

Solution. We will solve the pipe sizing problem in 8 steps:

1. Since the pipe size will be measured in inches, we will assume the following basic conversion units:
 - 1 mile = 5280 ft
 - 600 miles = $600 \times 5280 = 3,168,000$ ft
 - 1 barrel = 42 U.S. gal
 - 1 barrel = $42 \text{ gal} \times 231 \text{ in.}^3/\text{gal} = 9702 \text{ in.}^3$
 - 1 barrel = $9702 \text{ in.}^3/12^3 = 5.6146 \text{ ft}^3$
 - Density of steel = 490.75 lb/ft^3
2. To determine the operating cost in pumping oil, we first need to determine power (electricity) required to pump oil:
 - Volume flow rate per hour:

$$\begin{aligned} Q &= 3,000,000 \text{ barrels/day} \times 5.6146 \text{ ft}^3/\text{barrel} \\ &= 16,843,800 \text{ ft}^3/\text{day} \\ &= 701,825 \text{ ft}^3/\text{hr} \end{aligned}$$

- Pressure drop:

$$\begin{aligned} \Delta P &= \frac{128QuL}{g\pi D^4} \\ &= \frac{128 \times 701,825 \times 8500 \times 3,168,000}{32.2 \times 12,960,000 \times 3.14159 D^4} \\ &= \frac{1,845,153,595}{D^4} \text{ lb/ft}^2 \end{aligned}$$

- Pumping power required to boost the pressure drop:

$$\begin{aligned} \text{power} &= \frac{1.333Q\Delta P}{1,980,000} \\ &= \frac{1.333 \times 701,825 \times \frac{1,845,153,595}{D^4}}{1,980,000} \\ &= \frac{871,818,975}{D^4} \text{ hp} \end{aligned}$$

- Power cost to pump oil:

$$\begin{aligned} \text{power cost} &= \frac{871,818,975}{D^4} \text{ hp} \times \$0.015/\text{hp.hr} \times 24 \text{ hr/day} \times 365 \text{ days/year} \\ &= \frac{\$114,557,013,315}{D^4} / \text{year} \end{aligned}$$

3. Pump and motor cost calculation

$$\begin{aligned}\text{pump and motor cost} &= \frac{871,818,975}{D^4} \times \$195/\text{hp} \\ &= \frac{\$170,004,700,125}{D^4}\end{aligned}$$

4. Required amount and cost of steel

$$\text{cross-sectional area} = \frac{3.14159[(0.51D)^2 - (0.50D)^2]}{4}$$

$$= 0.032D^2 \text{ ft}^2$$

$$\text{total volume of pipe} = 0.032D^2 \text{ ft}^2 \times 3,168,000 \text{ ft}$$

$$= 101,376D^2 \text{ ft}^3$$

$$\text{total weight of steel} = 101,376D^2 \text{ ft}^3 \times 490.75 \text{ lb/ft}^3$$

$$= 49,750,272D^2 \text{ lb}$$

$$\text{total pipeline cost} = \$1.00/\text{lb} \times 49,750,272D^2 \text{ lb}$$

$$= \$49,750,272D^2$$

5. Annual equivalent cost calculation

$$\text{capital cost} = \left(\$49,750,272D^2 + \frac{\$170,004,700,125}{D^4} \right) (A/P, 10\%, 20)$$

$$= 5,843,648D^2 + \frac{19,968,752,076}{D^4}$$

$$\text{annual power cost} = \frac{\$114,557,013,315}{D^4}$$

6. Economical pipe size

$$AE(10\%) = 5,843,648D^2 + \frac{19,968,752,076}{D^4} + \frac{\$114,557,013,315}{D^4}$$

To find the optimal pipe size (D) that results in the minimum annual equivalent cost, we take the first derivative of AE(10%) with respect to D, equate the result to zero, and solve for D.

$$\frac{dAE(10\%)}{dD} = 11,687,297D - \frac{538,103,061,567}{D^5}$$

$$= 0$$

$$11,687,297D^6 = 538,103,061,567$$

$$D^6 = 46,041.70$$

$$D^* = \mathbf{5.9868 \text{ ft}}$$

Note that velocity in a pipe should be ideally no more than around 10 ft/sec due to friction wearing in the pipe. To check to see if the answer is reasonable, we may compute

$$Q = \text{velocity} \times \text{pipe inner area}$$

$$701,825 \text{ ft}^3/\text{hr} \times \frac{1}{3,600} \text{ hr/sec} = V \frac{3.14159(5.9868)^2}{4}$$

$$V = 6.93 \text{ ft/sec}$$

which is less than 10 ft/sec. Therefore, the optimal answer as calculated can be practical.

7. Equivalent annual cost at optimal pipe size

- Capital cost:

$$\begin{aligned} \text{capital cost} &= \left[\$49,750,272(5.9868)^2 + \frac{170,004,700,125}{5.9868^4} \right] (A/P, 10\%, 20) \\ &= 5,843,648(5.9868)^2 + \frac{19,968,752,076}{5.9868^4} \\ &= \mathbf{\$224,991,039} \end{aligned}$$

- Annual power cost:

$$\begin{aligned} \text{annual power cost} &= \frac{114,557,013,315}{5.9868^4} \\ &= \mathbf{\$89,174,911} \end{aligned}$$

- Total annual equivalent cost:

$$\begin{aligned} \text{total annual cost} &= \$224,991,039 + \$89,174,911 \\ &= \mathbf{\$314,165,950} \end{aligned}$$

8. Total annual oil revenue

$$\begin{aligned} \text{annual oil revenue} &= \$18/\text{bbl} \times 3,000,000 \text{ bbls/day} \times 365 \text{ days/year} \\ &= \$19,710,000,000/\text{year} \end{aligned}$$

There are enough revenues to offset the capital as well as operating cost.

Comments. A variety of other criteria exists for choosing pipe size for a particular fluid transfer application. For example, low velocity may be required where there are erosion or corrosion concerns. Alternatively, higher velocities may be desirable for slurries where setting is a concern. Constructional ease will also weight significantly in the choice of pipe size. A small pipe size may not accommodate the head and flow requirements efficiently, whereas space limitations may prohibit selecting large pipe size.

17.7 Project Management

Donald D. Tippett

Engineers, Projects, and Project Management

For a majority of engineers, project work is a way of life throughout their professional careers. Projects build bridges, put men in space, design and install computer-integrated manufacturing systems, perform R&D on state-of-the-art materials, and pursue a thousand other objectives in organizations large and small across the world.

A project has a specific set of objectives and a definite schedule, budget, and set of performance requirements. It has a finite life span and is usually a team effort led by a recognized project manager.

Project management is planning, organizing, leading, and controlling organizational resources to successfully complete a project.

Project Planning

The two most important facets of project management are planning and team building (to be discussed in the section on team building).

Project Charter. The first step in a sound planning process is the issuance of a project charter. A project charter is a formal establishing of the project by the upper management chartering authority (a project is not really a project unless it has management's sanction). As a minimum, the charter should: (1) designate the purpose of the project, (2) establish the general organizational format for the project, (3) appoint a project manager, and (4) state management's support for the project.

All projects should be formally launched with some sort of charter. For small in-house projects, it may be only a brief memo. Larger projects will have much more extensive charters. A charter puts the organization on notice that the project is being launched and that it has management's acknowledgment and commitment. The charter is the project manager's action authority to perform the project.

What is Project Planning?

- Planning is determining what needs to be done, by whom, and by when.
- Planning is decision making based upon futurity.
- Planning is selecting enterprise objectives, and establishing policies/procedures/programs necessary for achieving them.
- Planning is an iterative process and must be performed throughout the life of the project (Kerzner, 1998).

Each of these statements adds to an understanding of what project planning is. But why plan? There are a number of reasons:

- The planning process serves as a simulation of the project in the planners' minds before significant resources are expended. Thus false starts are avoided and problems are identified and resolved when solutions are the easiest.
- The planning process serves as a means of communications, informing everyone on the project what will be expected of them.
- Plans identify inconsistency and risk.
- Plans provide the basis for action.

Plans must be flexible, and provisions should be provided to update them as needed throughout the project. They should include reasonable contingencies and usually require several iterations before they are acceptable.

Whenever possible, the people who will do the work should plan the work. This develops a sense of ownership of the tasks among those who will actually execute them (Rosenau, 1992).

Senior managers tend to apply pressure on the project manager to cut the planning process prematurely short. However, cutting the planning process short is usually false economy. As much as 50% of direct labor hours and dollars can be spent before project execution begins, and companies that spend significantly less usually find quality problems during execution (Kerzner, 1998).

Project Planning Steps. The steps in sound project planning are:

- Establish objectives
- Develop [statement of work](#)
- Prepare detailed specifications
- Establish project milestones
- Create the [work breakdown structure](#)
- Establish detailed schedules
- Establish detailed budgets
- Define responsibilities
- Replan as required

Objectives. Project objectives are the basis for all project planning. Objectives must be clearly stated, specific, measurable, and achievable. They should also be realistic and agreed upon by all parties concerned. Objectives should be consistent with the resources available and with other organizational plans and projects.

Statement of Work/Specifications. The statement of work (SOW) specifically states what the project will do for the customer. For large projects it will be a formal part of the contract. For projects within the organization it might be a memorandum. The SOW specifies the reason for establishing the project, the desired end results, and the performance, budget, and schedule goals. It may also include specific acceptance criteria, as well as a management section which discusses client relationships, management philosophies, project organization and personnel reporting, contingencies, and communications. The establishment of a satisfactory statement of work may require several iterations. However, it is critical that all parties have a detailed understanding of what is to be done before work proceeds.

Depending upon the size of the project, detailed specifications may be listed separately, or included as part of the SOW. They define the standards which must be met with respect to materials, labor, prices, equipment, support, etc. It is important to firmly establish detailed specifications up front, as small changes in specifications can have major effects on budgets and schedules.

Project Milestones. Project milestones establish the start and end dates of the project (if known), as well as all the significant subsets. Milestones are designated points in time by which certain specific project tasks/accomplishments are to be completed. The set of milestones forms a group of waypoints which provide a basis for status assessment, management reviews, and replanning the project. Because they are simple to understand, they are an important tool to gauge progress.

Work Breakdown Structure. The work breakdown structure (WBS) is a systematic way of defining the component parts of the project. The WBS breaks down the work to be accomplished into a logical hierarchy of sets, subsets, etc., so that it is put into “bite-sized” chunks which can be easily understood by planners, schedulers, workers, and managers. More specifically, breaking the project’s work into manageable pieces:

- Facilitates the planning process by describing the total job in terms of discrete tasks.
- Allows assignment of these tasks to specific groups and individuals.
- Forms the framework upon which the budget and schedule are built, since each WBS task can easily be estimated in terms of labor, duration, and other required resources.

- Facilitates the chore of scheduling, supervision, and project control and information systems by defining exactly who is responsible for each task and establishing a target completion time.
- Simplifies purchasing, scheduling, and staging of required materials.

In creating a WBS, the following principles should be kept in mind:

- Routine or repetitive work should not be excessively subdivided.
- Subdivide each block of work to a level which is useful to project management; all blocks do not have to have the same number of subdivisions.
- Each task should be easily understood and should have an identifiable schedule.
- The cost of the smallest subtask should be significant.

It is essential that a work breakdown structure be created. The WBS forms the skeleton upon which the plan, schedule, budget, and control system are built. In addition, a completed WBS is the basis of entering project information into any of the myriad of computerized planning and scheduling systems now being used by virtually all project managers.

Scheduling/Budgeting/Responsibilities/Replanning. The remaining components of project planning — developing a schedule and budget, and assigning responsibilities — naturally follow the creation of the WBS. Once the project's work has been separated into a set of manageable subtasks, it is a straightforward process to set about estimating the time and resources required to complete each one (see the section on estimating). These estimates are then compiled into a project budget. Similarly, responsibility for each WBS component can be assigned to individuals and groups as appropriate and compatible with available resources. While the scheduling process involves a few more intermediate steps (see the next section), it too is a logical extension of the work breakdown structure. Much of the project plan depends on the WBS. Therefore project managers should pay close attention to it.

Since plans relate to future events but are based upon current knowledge, it follows that plans will need to change during the course of the project. As the project team learns lessons along the way, and as events unfold in the dynamic project environment, project plans will need to be adjusted accordingly. Vehicles for accomplishing this process should be established at the outset. Plans must be kept current. Out-of-date plans hurt project credibility. Besides, the project team needs an accurate roadmap if it is to arrive successfully at its desired destination.

Project Scheduling

Creating a viable schedule and adjusting it as necessary throughout the life of the project is an important part of the project manager's responsibilities.

The scheduling cycle (see [Figure 17.7.1](#)) begins with the work breakdown structuring process, which subdivides the project's work into a set of manageable and easily understood tasks. The next step is to estimate the time, man hours, and other resources required to complete each task. Then the relationships between tasks must be documented. Many can be performed in parallel while some must necessarily come before others. In any case, each task's interdependence with every other task must be determined. The project manager is now ready to create the actual schedule, using one of the methods discussed in the next section. Once the schedule is formulated, it must be checked against available resources for feasibility. If the schedule contains periods of time during the project when more resources are called for than will be available, the scheduler must iterate back to an earlier part of the scheduling cycle and make adjustments so that the eventual schedule will be compatible with available resources. Once this is accomplished, the project manager should make a final check to ensure that the proposed schedule will in fact satisfy the project constraints (budget, time, and performance criteria). If constraints are met, the schedule can be finalized.

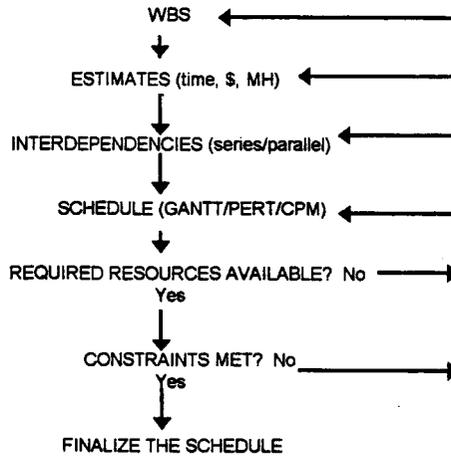


FIGURE 17.7.1 Project scheduling and estimating cycle.

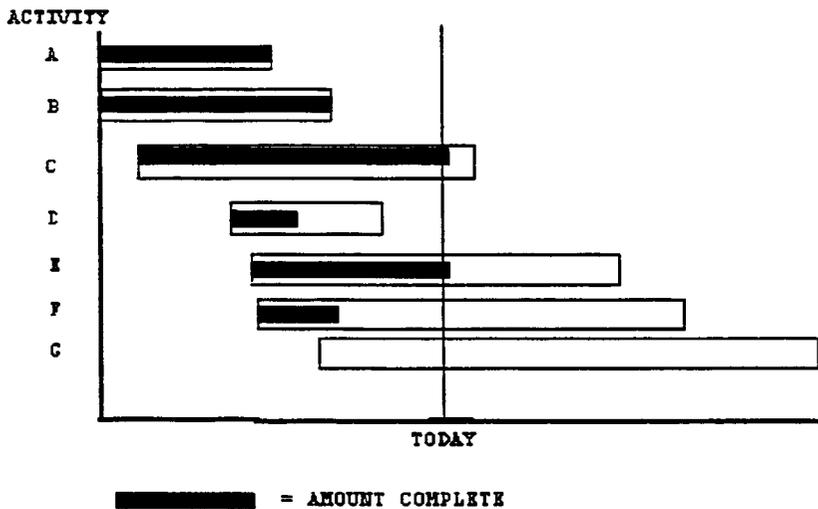


FIGURE 17.7.2 Gantt chart example.

Scheduling is a continuous process during the life of a project. In the dynamic project environment, changes are always occurring which necessitate replanning and rescheduling. Mechanisms that keep the project plan and schedule valid and current should be put in place at the outset of the project.

Scheduling Methods. The [Gantt chart](#) or bar chart (see [Figure 17.7.2](#)) was one of the first techniques used by project managers to schedule and control projects, and it is still widely used. It is simple, graphic, easy to construct and change, and convenient for displaying progress. Project task times are usually shown as horizontal bars that begin and end at the scheduled activity start and end times, respectively. However, Gantt charts do not do a good job of displaying task dependencies and interrelationships. Furthermore, the critical path, the set of sequential activities which define the ultimate length of the project, is not obvious (Rosenau, 1992). So, while the Gantt chart is a useful tool for quick overviews and broad statusing of projects, it needs to be augmented by a more comprehensive scheduling tool in all but the most elementary project management situations.

Critical path method (CPM) and program evaluation and review technique (PERT) are two critical path systems developed to address the shortcomings of the Gantt chart, and enable project managers to

do a better job scheduling complex projects. They both show project task durations, and indicate the earliest task start times as the Gantt chart does, but they also show the *precedence* between activities. This allows the project manager to determine the most critical activities and to more easily modify and determine the status of the schedule.

CPM assumes activity durations are known with a reasonable degree of certainty, and are thus useful in industries like construction that have a long history of performing the various tasks to draw from in formulating accurate project activity estimates (e.g., dig basement, pour footer, etc.). On the other hand, PERT is designed to be used in applications like R&D and aerospace wherein many activities are being performed for the first time and there is no history to draw upon in formulating activity duration estimates. PERT employs statistical techniques to assist the scheduler to devise reasonable activity duration estimates and an estimate for the overall project duration.

Figure 17.7.3 depicts a simple CPM network drawn in the activity on node (AON) format. The nodes or circles denote activity. The connecting arrows show precedence between activities. There are various possible routes through the network. The longest route defines the predicted total project duration. The activities on this longest route, or critical path, are called critical activities. Any delay in the execution of these critical activities translates into a delay in overall project completion. Other non-critical activities have a certain amount of leeway (or slack) associated with them which allows the project manager some flexibility in scheduling resources. Thus the project manager should pay special attention to critical activities in making tradeoff decisions, as these are the ones that can potentially extend the project duration. However, it should be noted that non-critical activities can potentially become critical if they experience delays greater than their original slack, so the project manager cannot become complacent about noncritical activities either.

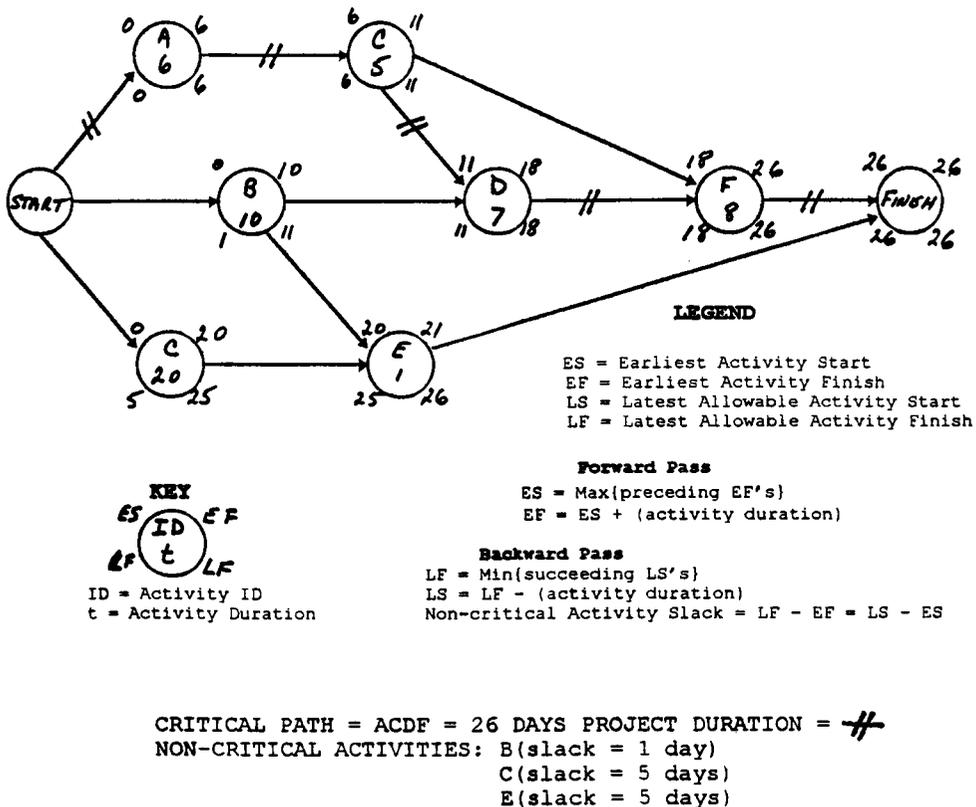


FIGURE 17.7.3 Example of AON CPM network.

In solving the network, forward and backward passes are made according to the formulas given in Figure 17.7.3. Together these two passes determine the earliest and latest starts and finishes for each activity. Also determined are the project's duration, and the critical path (path that contains activities without slack), which represents the predicted project duration. In the example, activities A, C, D, and F have no slack and are thus critical. The path ACEF is the critical path and defines the project duration, 26 days. Once created, the network can be continually updated and used during the life of the project to assist the project manager to understand current project status and make informed tradeoff decisions.

It is not feasible to solve any but the most simple PERT/CPM networks by hand. Fortunately, software is widely available for modern personal computers which is capable of constructing and solving these networks for small and medium sized projects. Very large project networks require the power of mainframe computers. The Dreger (1992) text presents comprehensive examples using typical personal computer project management software in realistic project situations.

A more detailed exposition of PERT/CPM techniques can be found in Moder et al., 1983.

Staffing and Organizing

In staffing and organizing a project, management must bring together a diverse group of people, equipment, information, and other resources that, when properly integrated, is capable of achieving the project objectives. Each element of the resulting system is critical to overall project success.

Selecting the Project Manager. The project manager is the most critical selection. A person should be selected who:

- Serves full time on the project. With few exceptions, appointing a part-time project manager is not practical for projects of any significance. Naming a part-time project manager signals the organization that the project is a low priority. This practice is not fair to the individual or to the project, and usually produces less than optimum results.
- Has a proven track record. Most project situations are poor training grounds for project managers with little or no experience or training. A project manager's credibility with the organization is vital.
- Functions as a manager, not a doer. If the project manager spends all of his or her time performing technical tasks, then, in effect, there is no project manager. With no one in the driver's seat, budgets, schedules, customer relations, and other critical project management components are left unattended.
- Possesses good interpersonal skills. A project manager may be called upon at any time to be a communicator, mediator, negotiator, motivator, coach, counselor, and leader.

Project Team Staffing. The project manager may have little or no control over the composition of the project team, as the functional managers of the contributing departments are often the ones who decide which individuals serve on the project. Hence, project managers must be able to mold a potentially broad mix of talents, functional backgrounds, and professional experience into an integrated team. This is one reason why team building skills are vitally important. Given a choice, the project manager would opt for project team members who are assigned full-time to the project, experienced in project work and their discipline, are team players, and relate well with customer representatives when direct interchange is required.

Organizing for Project Management. Projects are conducted under an infinite variety of organizational structures. Project managers usually have little input in this area and must learn to make the best of the organizational structure in which they find their project operating. However, project managers should bear in mind that, while organizational structures may serve either to facilitate or impede project progress, they are usually not a principal factor in determining overall project success or failure. Other factors like thorough planning and building a cohesive, motivated project team are more predictive of project

TABLE 17.7.1 Advantages and Disadvantages of the Three Main Organizational Forms from a Project Management Perspective

Organizational Structure	Advantages	Disadvantages
Functional/Hierarchical/ Classical	<ul style="list-style-type: none"> * Close control of budget and people. * Well-defined authority and communications channels. * Well-understood career paths. * Best for mass production. * Nurtures functional areas; assures up-to-date technology. * Provides security for individuals (in functional groups) 	<ul style="list-style-type: none"> * No one person truly in charge of the entire project. * Long lead times, ineffective integration, very slow internal communications. * Ineffective communications with the customer. * Difficult to establish status of projects. * Very slow to respond to change. * Strongest functional group tends to dominate decision making. * Poor conflict resolution, great reliance on hierarchical referral.
Pure Project/Product/ Divisional	<ul style="list-style-type: none"> * One individual in authority to speak for the project. * Quick response to changing environment. * Personnel allocated completely to the project * Good communications. * Less need for hierarchical referral to resolve conflict. * Most likely to complete the project on time. 	<ul style="list-style-type: none"> * Less efficient use of resources. * Does not nurture functional disciplines. * Best when company is project-oriented. * Uncertainty for personnel when project ends (where do they go?).
Matrix	<ul style="list-style-type: none"> * More efficient: project can share personnel and more easily accommodate changing requirements. * Good communications. * Personnel security (people have homes to go to after project is finished). * Technical disciplines are supported/nurtured. * Quick to respond to change. * One person speaks for the project. 	<ul style="list-style-type: none"> * High potential for conflict. * Project personnel must work for two bosses. * Heavy penalty in communications requirements and other administrative overhead costs. * Best for project-oriented organization. * Competition for best resources. Upper management may use matrix as requiring them to give up some authority.

success. Besides, as Kerzner (1998) so aptly points out, any organizational form will work if the people want it to.

Nevertheless, project managers should be cognizant of the strengths and weaknesses of the organizational structure under which they are operating so they can make the best of their situation. [Table 17.7.1](#) gives some of the advantages and disadvantages of the three principal organizational structures and their many variants, from a project management perspective.

Regardless of the organizational form used, it is a good idea to create a steering team to oversee and guide important projects. A steering team of senior managers demonstrates management support of the project, helps with integration and hard problems, provides oversight and guidance, makes major dollar decisions, and acts as an interface with upper management.

Team Building

Today’s projects characteristically require the efforts of many people from multiple disciplines whose efforts must be effectively integrated to meet project objectives. Further, project managers must cope with dynamic environments, high degrees of organizational complexity, and ever-increasing competitive pressures. Experienced project managers know that the only way to effectively deal with these challenges

is to mold their project personnel into true project *teams* or groups of people with complementary skills who are committed to a common purpose, set of performance goals, and approach for which they hold themselves mutually accountable (Katzenbach and Smith, 1993). Teams feature enhanced efficiency, increased motivation, self-regulation, synergistic output, flexibility, and heightened confidence (Raudsepp, 1984), all of which are vital to project success in the present competitive environment. Thus, team building is an essential project management skill.

Robert P. Hagen has advanced six key elements of most successful team building plans which, if implemented by project managers, would greatly enhance the state of team building on project teams (Table 17.7.2).

TABLE 17.7.2 Six Key Team Building Elements

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1. In all actions, demonstrate respect and consideration for all employees as valued members of the team. Are employees encouraged by example and admonition to respect each other? Do they know enough about each other's job to appreciate the contributions others are making? Does a general atmosphere of consideration exist?
 2. Identify individual job responsibilities and performance standards and see that they are known. Are individual discussions held to ensure that each employee knows their job's standards and responsibilities? Does each team member understand how their portion of the project is important to overall project success?
 3. Work to secure good communications with employees as individuals and as a team. Do team members feel their inputs and suggestions are valued? Do they receive regular feedback on how they are doing? Is advance warning of changes conveyed whenever possible along with reasons why changes are necessary? Are regular exchange meetings held? Are team members included in decision-making?
 4. Establish individual and group goals, preferably in coordination with those concerned. Are *individual* goals established for each team member? Is consideration given to each individual's opportunities for professional development? Are *group* goals established and communicated to the team? Is a goal established that encourages growth in team development factors like team planning, conflict resolution, and problem solving?
 5. Reward teamwork and team building efforts. Who issues rewards? Project managers? Functional managers? Are rewards mostly based on extrinsic job factors like pay, bonuses, and working conditions, or are they based on intrinsic job factors like accomplishment, recognition, responsibility, and growth? Does management recognize the difference between rewarding an individual as a member of a team, and rewarding an individual as an individual? Are individuals singled out and rewarded for their performance on the project team? Do team members have input into what and how rewards are given and to whom?
 6. Practice and encourage loyalty to the team. Does the project manager defend team members against unfair criticism? Is there a climate of trust on the project team? Is effective leadership practiced by project managers?
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Project Control

Planning puts the team in a position to launch a project. However, rarely does everything go according to plan. As soon as the project begins, deviations start to occur. Thus, once the project is launched, conforming to the plan becomes a principal function of project management.

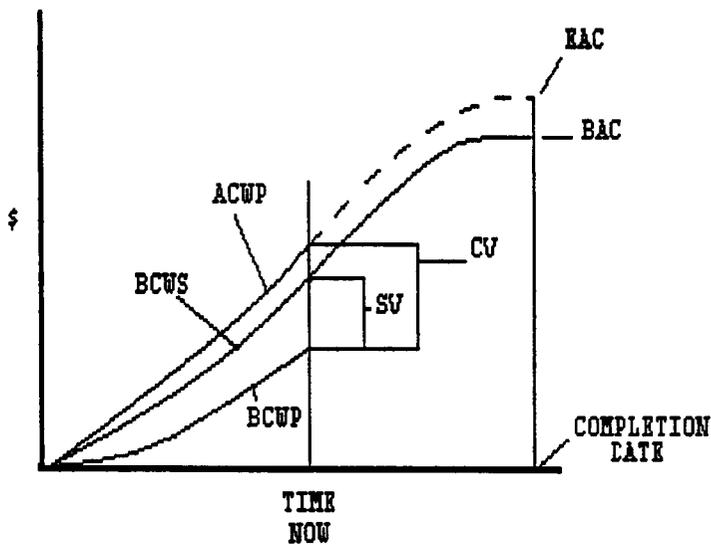
It is up to the project control system to keep the project manager apprised of how all components of the project are progressing, and highlight significant problems and deviations from plan, so that corrective action can be taken. Without a control system, the project manager has little influence over the project, and it will meander to completion in some form or fashion.

An effective project control system combines a cost accounting function and a cost control function with appropriate mechanisms for monitoring progress against schedule and technical performance targets. The cost accounting function accumulates actual costs, ensures costs are properly allocated, and verifies work is carried out and billed correctly. The cost control function provides the information to support cost analysis, prediction, and reporting. Costs are maintained within budget by taking corrective action wherever predicted costs exceed the budget.

Among the most critical requirements of a project control system is providing a capability to compare budgeted costs to actual costs, and then conduct variance analyses which lead to corrective action. The earned value approach to variance analysis is widely used. It compares the **budgeted cost of work performed** (BCWP), or earned value, with the **actual cost of work performed** (ACWP) to determine the cost variance (CV). It compares the BCWP with the **budgeted cost of work scheduled** (BCWS), or planned earned value, to determine the schedule variance (SV).

ACWP and BCWS are fairly straightforward to obtain. On the other hand, it may be difficult to obtain an accurate representation of BCWP. One approach is to use standard dollar expenditures for the project, so that x% of the costs are booked for x% of the time. Another is the 50/50 rule, wherein half the budget is recorded when an activity is scheduled to begin and the other half when the activity is scheduled to complete. Other alternative approaches may depend on percentage of man hours expended, parts installed, programs written/tested, etc. There are many other possible approaches; however the object is to somehow measure the percentage complete for project activities (or *earned value*). On some projects the time, trouble, and expense to obtain an accurate BCWP may not be cost effective.

Figure 17.7.4 gives a graphic presentation of the earned value approach and summarizes some quantitative indices useful to project managers conducting variance analysis. Once identified, each critical variance must be investigated to determine the cause and the appropriate corrective action. Such an investigation should begin at the lowest organizational level by the supervisor involved. It then progresses upward to the project manager, who reviews variance causes and approves corrective actions. Normally the project manager will review these results with upper-level management prior to preparing the contractually required reports to the customer.



- BCWP = BUDGETED COST OF WORK PERFORMED = EARNED VALUE
- BCWS = BUDGETED COST OF WORK SCHEDULED = PLANNED EARNED VALUE
- ACWP = ACTUAL COST OF WORK PERFORMED
- CV = COST VARIANCE = BCWP - ACWP
- SV = SCHEDULE VARIANCE = BCWP - BCWS
- CV% = COST VARIANCE PERCENTAGE = CV/BCWP
- SV% = SCHEDULE VARIANCE PERCENTAGE = SV/BCWS
- EAC = ESTIMATE AT COMPLETION = (ACWP/BCWP) * (TOTAL BUDGET)
- (NEGATIVE IS UNFAVORABLE)

FIGURE 17.7.4 Variance analysis indices and graphics.

Estimating and Contracting

Project Estimating. Estimating time and resource requirements on projects is frustrating because the project manager knows in advance that the probability of being exactly correct is slim. The key, then, is to not be afraid to be wrong, but rather simply strive to be as accurate as possible, given what is known at the time.

A simple three-step approach will serve as a starting point for estimating:

1. Top-Down Estimate — Make an initial estimate at an overview level...an order-of-magnitude estimate using the broad, big picture view.
2. Detailed Bottom-Up Estimate — Follow the top-down estimate with a detailed estimate built from the bottom up. Start with the lowest tasks in the work breakdown structure and work up by task, by department, and so on, arriving at a total figure.
3. Compare the two estimates.

In effect, each estimate provides a reality check for the other. If the two are in relative agreement, it is likely that a good estimate has been developed. However, if the two figures are very different, more analysis is indicated. Go back and examine the individual work packages to discover where costs may differ, and to what extent there have been incorrect assumptions made as to the scope and content of work called for. Once the areas of principal differences between the two estimates have been determined, decisions can be taken as to which is correct.

Contingency planning is an important part of estimating. It is usually not appropriate to place a blanket contingency figure (e.g., 10%) across all tasks. Rather, contingency budgets should be set deliberately considering each activity individually, being careful not to build contingency on top of contingency.

In developing detailed estimates, time and dollars to cope with the following issues should not be overlooked (Rosenau, 1992).

- Interfacing with customers, upper management, and other departments
- Obtaining customer furnished items
- Getting approvals
- Working at remote/difficult locations
- Training people
- Making mistakes
- Obtaining security clearances
- Placing major subcontractors and purchase orders
- Replacing sick and vacationing people

Contracts. There are many types of contractual agreements used on projects. However, most can be categorized as variations upon two basic types of contracts: fixed-price and cost-reimbursable. Under a fixed-price contract the customer agrees to pay a specific amount upon satisfactory completion of the project. A cost-reimbursable contract is one in which the customer agrees to reimburse the contractor organization for costs actually incurred in performing the project.

The fixed price contract presents the lowest financial risk to the customer because the maximum financial liability is specified. This form presents the highest risk to the contractor organization, though it also opens the door for increased profits if the contracted costs can be underrun. On the other hand, under a cost-reimbursable contract, the customer is obligated to reimburse the contractor for all costs incurred, and thus bears considerable risk. The usual arrangement under cost-reimbursable contracts is to add a fixed fee or an incentive fee to the costs to arrive at the total compensation figure due the contractor. Cost-reimbursable contracts are traditionally used when the nature of the project dictates that accurate pricing cannot be determined.

Market conditions also influence the type of contract used. When work is scarce, customer organizations often insist upon fixed-price contracts. Conversely, when business is good, customers are not able to insist upon fixed-price contracts, and more cost-reimbursable type contracts are used.

Summary

Project management is among the most challenging undertakings in the field of management today. Project managers routinely face formidable performance requirements, firm budget constraints, and tight schedules. They must organize people from diverse technical backgrounds into a cohesive, motivated team, and then lead that team as it plans and executes its complex tasks in an ever-changing environment.

While the most successful technical people are often promoted to project management, it takes more than a solid technical background to make an effective project manager. To qualify an individual for project management, sound training in the fundamentals is essential, along with on-the-job experience as a project team member and/or assistant project manager or project engineer.

The above constitutes but the tip of the iceberg in terms of required knowledge for project managers, but it will perhaps give the uninitiated a starting point. The excellent references listed below will serve to fill in many of the gaps for the interested reader.

Defining Terms

Activity on Node (AON): A scheduling network in which the circles (or nodes) represent activities, while the connecting lines denote the sequence the activities must follow (see Figure 17.8.3).

Actual Cost of Work Performed (ACWP): Actual dollars expended on a given project activity.

Budgeted Cost of Work Performed (BCWP): Earned value of completed work.

Budgeted Cost of Work Scheduled (BCWS): Planned earned value.

Gantt Chart: The Gantt chart, named after Henry L. Gantt, is a bar chart useful in project management to depict planned and actual progress of project activities on a horizontal time scale (see Figure 17.8.2).

Network: A combination of arcs and nodes linked together to represent project activity durations and precedences. Used in conjunction with a solution technique like CPM or PERT, estimates of project completion times, critical activities, and resource requirements can be determined.

PERT/CPM: PERT stands for program evaluation and review technique, while CPM stands for critical path method. Both are methods for solving a project activity network. CPM is used on projects where activity time estimates are well established, like on construction jobs. PERT is used on projects where times to complete activities are not known with a great deal of certainty, such as in aerospace and R&D applications. Almost always used in conjunction with computer software, these techniques yield estimates of overall project completion times, listings of the most critical activities, and resource requirements by time frame.

Statement of Work (SOW): Formal statement delineating specifically what the project will do for the customer. As a minimum it should contain performance, budget and schedule objectives.

Work Breakdown Structure (WBS): A logical division of the work involved in the project into a hierarchy of component parts. It forms the basis for subsequent project budgeting, scheduling, and controlling.

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