DUALITAS DAN ANALISIS SENSITIVITAS

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Primal Problem P

minimize $z = cx$

subject to $Ax = b$

$x \geq 0$

optimum value is z^*

Dual Problem D

maximize $v = \pi b$

subject to $\pi A \leq c$

optimum value is v^*

Theorem. (Strong Duality) If both P and D are feasible, then $z^* = v^*$.

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Tabel Primal – Dual Linear Programming

			PRIMAL					
			Koefisien				NK	
			X_1	X_2	.....	X_n		
DUAL	koefisien	Y_1	a_{11}	a_{12}	.....	a_{1n}	$\leq b_1$	Koefisien fungsi tujuan (maksimisasi)
		Y_1	a_{11}	a_{12}	.....	a_{1n}	$\leq b_2$	
		....	....	.....	.....	.....		
		Y_m	a_{m1}	a_{m2}	.....	a_{mn}	$\leq b_1$	
	NK		$\geq C_1$	$\geq C_2$	.....	$\geq C_n$		
			Koefisien fungsi tujuan (maksimisasi)					

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Tabel Hubungan antara primal - dual

Primal (atau Dual)	**Dual (atau Primal)**
Batasan I	Variabel I
Fungsi tujuan	Nilai kanan

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The min cost flow problem and its dual

Minimize $\sum_{(i,j)\in A} c_{ij}x_{ij}$

$\sum_j x_{ij} - \sum_k x_{ki} = b_i$ **for all i Î N.**

and $x_{ij} \geq 0$ **for all (i,j) Î A.** **Primal**

Minimize $\sum_{i=1}^{n} \pi_i b_i$ **Dual**

subject to $\pi_i - \pi_j \leq c_{ij}$ **for all** $(i,j) \in A$

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MASALAH PRIMAL

MAX : $Z = 3X_1 + 5X_2$

S.T.: $2X_1 \leq 8$

$3X_2 \leq 15$

$6X_1 + 5X_2 \leq 30$

$X_1 >= 0$

$X_2 >= 0$

MASALAH DUAL

MIN : $Y = 8Y_1 + 15Y_2 + 30Y_3$

S.T.: $2Y_1 + 6Y_3 \geq 3$

$3Y_2 + 5Y_3 \geq 5$

$Y_1 \geq 0$

$Y_2 \geq 0$

$Y_3 \geq 0$

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PENYELESAIAN PRIMAL :

OBJECTIVE FUNCTION VALUE

1) 27.5000000

VARIABLE	VALUE	REDUCED COST
X1	.833333	.000000
X2	5.000000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	6.333333	.000000
3)	.000000	.833333
4)	.000000	.500000

Kendala aktif

PENYELESAIAN DUAL

OBJECTIVE FUNCTION VALUE

1) 27.5000000

VARIABLE	VALUE	REDUCED COST
Y1	.000000	6.333333
Y2	.833333	.000000
Y3	.500000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	-.833333
3)	.000000	-5.000000

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PENYELESAIAN TABEL DENGAN PRIMAL

variabel dasar	Z	X_1	X_2	X_3	X_4	X_5	NK	Keterangan
Z	1	- 3	- 5	0	0	0	0	
X_3	0	2	0	1	0	0	8	
X_1	0	0	3	0	1	0	15	
X_5	0	6	5	0	0	1	30	

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variabel dasar	Z	X_1	X_2	X_3	X_4	X_5	NK	Keterangan
Z	1	-3	-5	0	0	0	0	
X_3	0	2	0	1	0	0	8	8/0 = ~
X_1	0	0	3	0	1	0	15	15/3= **5**
X_5	0	6	5	0	0	1	30	30/5 = 6
variabel dasar	Z	X_1	X_2	X_3	X_4	X_5	NK	Keterangan
Z	1	-3	0	0	5/3	0	25	
X_3	0	2	0	1	0	0	8	
X_2	0	0	1	0	1/3	0	5	
X_5	0	6	0	0	-5/3	1	5	

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variabel dasar	Z	X_1	X_2	X_3	X_4	X_5	NK	Keterangan
Z	1	-3	0	0	5/3	0	25	
X_3	0	2	0	1	0	0	8	8/2 = 4
X_1	0	0	1	0	1/3	0	5	5/0 = ~
X_5	0	6	0	0	-5/3	1	5	5/6 = **5/6**
variabel dasar	Z	X_1	X_2	X_3	X_4	X_5	NK	Keterangan
Z	1	0	0	0	5/6	1/2	27½	nilai optimal
X_3	0	0	0	1	5/9	-1/3	6⅓	
X_1	0	0	1	0	1/3	0	5	
X_5	0	1	0	0	-5/18	1/6	5/6	

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Apabila batasan 1 : $3X_2 \leq 15$ dirubah menjadi $3X_2 \leq 16$ nilai nya akan tetap

$$6X_1 + 5X_2 = 30 \rightarrow \times 3$$
$$3X_2 = 16 \rightarrow \times 5$$
$$18X_1 + 15X_2 = 90$$
$$15X_2 = 80$$
$$18X_1 = 10$$
$$X_1 = \frac{10}{18} = \frac{5}{9} = 0,56$$
$$6\left(\frac{5}{9}\right) + 5X_2 = 30$$
$$3,3 + 5X_2 = 30$$
$$5X_2 = 30 - 3,3$$
$$X_2 = \frac{26,7}{5} = 5,34$$

$$Z = (3 \times 0,56) + (5 \times 5,34)$$
$$Z = 1,67 + 26,7 = 28,37$$
$$\Delta Z = 28,37 - 27,5 = 0,87$$

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Apabila batasan 3 : $6X_1 + 5X_2 \leq 30$ dirubah menjadi $6X_1 + 5X_2 \leq 31$

$$6X_1 + 5X_2 = 31 \rightarrow \times 3$$
$$3X_2 = 15 \rightarrow \times 5$$
$$18X_1 + 15X_2 = 93$$
$$15X_2 = 75$$
$$18X_1 = 18$$
$$X_1 = \frac{18}{18} = 1$$
$$6(1) + 5X_2 = 31$$
$$6 + 5X_2 = 31$$
$$5X_2 = 25$$
$$X_2 = \frac{25}{5} = 5$$

$$Z = (3 \times 1) + (5 \times 5)$$
$$Z = 3 + 25 = 28$$
$$\Delta Z = 28 - 27,5 = 0,5$$

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Hubungan antara variabel-variabel Primal-Dual dalam Linear Programming

Variabel Primal	Variabel Dual
Variabel asli : X_1	Variabel surplus : $Z_j - C_j$
Variabel Slack : X_{n+i}	Variabel Asli : Y_i
	Dimana i = 1,2, … m
	j = 1,2, … n

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PENYIMPANGAN-PENYIMPANGAN DARI BENTUK STANDAR

Konversi Bentuk Bukan Standar Menjadi Bentuk Standar Dalam Model Linear Programming

Bentuk bukan standar :

Minimisasi Z

$$\sum_{j=1}^{n} a_{ij} X_j \geq b_i$$

$$\sum_{j=1}^{n} a_{ij} X_j = b_i$$

Nilai X_j tidak terbatas

Bentuk standar :

Maksimisasi Z

$$\sum_{j=1}^{n} a_{ij} X_j \leq -b_i$$

$$\sum_{j=1}^{n} a_{ij} X_j \leq b_i$$

$$-\sum_{j=1}^{n} a_{ij} X_j \leq -b_i$$

$$(X'_j - X''_j),\ X'_j \geq 0,\ X''_j \geq 0$$

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Mencari bentuk dual dari suatu masalah dual

Minimisasi $Y_0 = b_1 Y_1 + b_2 Y_2 + ... + b_m Y_m$

s / t :

$a_{11}Y_{11} + a_{21}Y_{21} + ... + a_{m1}Y_{m1} \geq c_1$

$a_{12}Y_{12} + a_{21}Y_{22} + ... + a_{m2}Y_{m1} \geq c_2$

$a_{1n}Y_{1n} + a_{2n}Y_{2n} + ... + a_{mn}Y_{mn} \geq c_n$

dan $Y_1 \geq 0; Y_2 \geq 0, ... Y_m \geq 0$

Perubahan ke dalam benuk standar

Maksimisasi $(-Y_0) = -b_1 Y_1 - b_2 Y_2 - ..... - b_m Y_m$

s / t :

$-a_{11}Y_{11} - a_{21}Y_{21} - ..... - a_{m1}Y_{m1} \leq -c_1$

$-a_{12}Y_{12} - a_{21}Y_{22} - ..... - a_{m2}Y_{m1} \leq -c_2$

$-a_{1n}Y_{1n} - a_{2n}Y_{2n} - ..... - a_{mn}Y_{mn} \leq -c_n$

dan $Y_1 \geq 0; Y_2 \geq 0, ... Y_m \geq 0$

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Dual dari dual tersebut (primal)

Minimisasi $(-Z) = -C_1X_1 - C_2X_2 - ..... - C_nX_n$

s / t :

$-a_{11}X_{11} - a_{21}X_{21} - ..... - a_{n1}X_{n1} \geq -b_1$

$-a_{12}X_{12} - a_{21}X_{22} - ..... - a_{n2}X_{n1} \geq -b_2$

$-a_{1m}Y_{1m} - a_{2m}Y_{2m} - ..... - a_{mn}Y_{mn} \geq -b_n$

dan $X_1 \geq 0; X_2 \geq 0, ... X_m \geq 0$

Perubahan ke dalam bentuk standar

Minimisasi $Z = C_1X_1 + C_2X_2 + ... + C_nX_n$

s / t :

$a_{11}X_{11} + a_{21}X_{21} + ... + a_{n1}X_{n1} \leq b_1$

$a_{12}X_{12} + a_{21}X_{22} + ... + a_{n2}X_{n1} \leq b_2$

$a_{1m}Y_{1m} + a_{2m}Y_{2m} + ... + a_{mn}Y_{mn} \leq b_n$

dan $X_1 \geq 0; X_2 \geq 0, ... X_m \geq 0$

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Dual dari suatu masalah dual tidak lain adalah masalah primalnya.

Batasan yang mengandung tanda persamaan (=) diperilukan seperti layaknya batasan bertanda ≤; tetapi batasan non-negatif bagi dual variabel yang bersangkutan harus dihilangkan (yaitu variabel yang tidak terbatas nilainya).

Menghilangkan batasan non-negatif pada masalah primal akan mengakibatkan perubahan batasan pada masalah dual menjadi bentuk persamaan (=)

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Hubungan bentuk-bentuk Primal - Dual

Masalah Primal (Dual)	Masalah Dual (Primal)
Max Z (atau Y_0)	Min Y_0 (atau Z)
Batasan i	Variabel X_j (atau Y_j)
bentuk ≤	$Y_j \geq 0$
bentuk =	$Y_j \geq 0$ dihilangkan
Variabel X_j (atau Y_j)	Batasan j
$X_j \geq 0$	bentuk ≥
$X_j \geq 0$ dihilangkan	bentuk =

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PRIMAL PROBLEM:

maximize $z = 3x_1 + 4x_2 + 6x_3 + 8x_4$
subject to
$x_1 + x_2 + x_3 + x_4 = 1$
$2x_1 + 3x_2 + 4x_3 + 5x_4 = 3$
$x_1, x_2, x_3, x_4 \geq 0$

DUAL PROBLEM:

minimize $y_1 + 3y_2$
Subject to
$y_1 + 2y_2 \geq 3$
$y_1 + 3y_2 \geq 4$
$y_1 + 4y_2 \geq 6$
$y_1 + 5y_2 \geq 8$

Observation 1.

The constraint matrix in the primal is the transpose of the constraint matrix in the dual.

Observation 2.

The RHS coefficients in the primal become the cost coefficients in the dual.

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PRIMAL PROBLEM:

maximize $z = 3x_1 + 4x_2 + 6x_3 + 8x_4$
subject to
$x_1 + x_2 + x_3 + x_4 = 1$
$2x_1 + 3x_2 + 4x_3 + 5x_4 = 3$
$x_1, x_2, x_3, x_4 \geq 0$

Observation 3. The cost coefficients in the primal become the RHS coefficients in the dual.

DUAL PROBLEM:

minimize $y_1 + 3y_2$
Subject to
$y_1 + 2y_2 \geq 3$
$y_1 + 3y_2 \geq 4$
$y_1 + 4y_2 \geq 6$
$y_1 + 5y_2 \geq 8$

Observation 4. The primal (in this case) is a max problem with equality constraints and non-negative variables

The dual (in this case) is a minimization problem with ≥ constraints and variables unconstrained in sign.

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OBJECTIVE FUNCTION VALUE
1) 4.66666700

VARIABLE	VALUE	REDUCED COST
X1	.666667	.000000
X2	.000000	.666667
X3	.000000	.333333
X4	.333333	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	-.333333
3)	.000000	1.666667

OBJECTIVE FUNCTION VALUE
1) 4.66666700

VARIABLE	VALUE	REDUCED COST
X1	.666667	.000000
X2	.000000	.666667
X3	.000000	.333333
X4	.333333	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	-.333333
3)	.000000	1.666667

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Analisis Sensitivitas

Karena terjadi perubahan-perubahan dalam variabel-variabel, apakah fungsi tujuan maupun fungsi kendala dengan cara memanfaatkan kaidah-kaidah primal-dual metode simplek semaksimal mungkin.

Karena tujuannya adalah penyelesian optimal, maka analisis ini disebut pula *Post Optimality.*

Perubahan-perubahan yang mungkin terjadi:

1. Keterbatasan kapasitas sumber (fungsi batasan).
2. Koefisien-koefisien fungsi tujuan.
3. Koefisien-koefisien teknis fungsi batasan
4. Penambahan variabel-variabel baru.
5. Penambahan batasan-batasan baru

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OBJECTIVE FUNCTION VALUE

1) 27.5000000

VARIABLE	VALUE	REDUCED COST
X1	.833333	.000000
X2	5.000000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	6.333333	.000000
3)	.000000	.833333
4)	.000000	.500000

SENSITIVITY ANALYSIS

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	3.000000	3.000000	3.000000
X2	5.000000	INFINITY	2.500000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	8.000000	INFINITY	6.333333
3	15.000000	3.000000	11.400000
4	30.000000	19.000000	5.000000

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OBJECTIVE FUNCTION VALUE

1) 27.5000000

VARIABLE	VALUE	REDUCED COST
Y1	.000000	6.333333
Y2	.833333	.000000
Y3	.500000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	-.833333
3)	.000000	-5.000000

SENSITIVITY ANALYSIS?

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
Y1	8.000000	INFINITY	6.333333
Y2	15.000000	3.000000	11.400000
Y3	30.000000	19.000000	5.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	3.000000	3.000000	3.000000
3	5.000000	INFINITY	2.500000

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